Spin relaxation in carbon nanotube quantum dots

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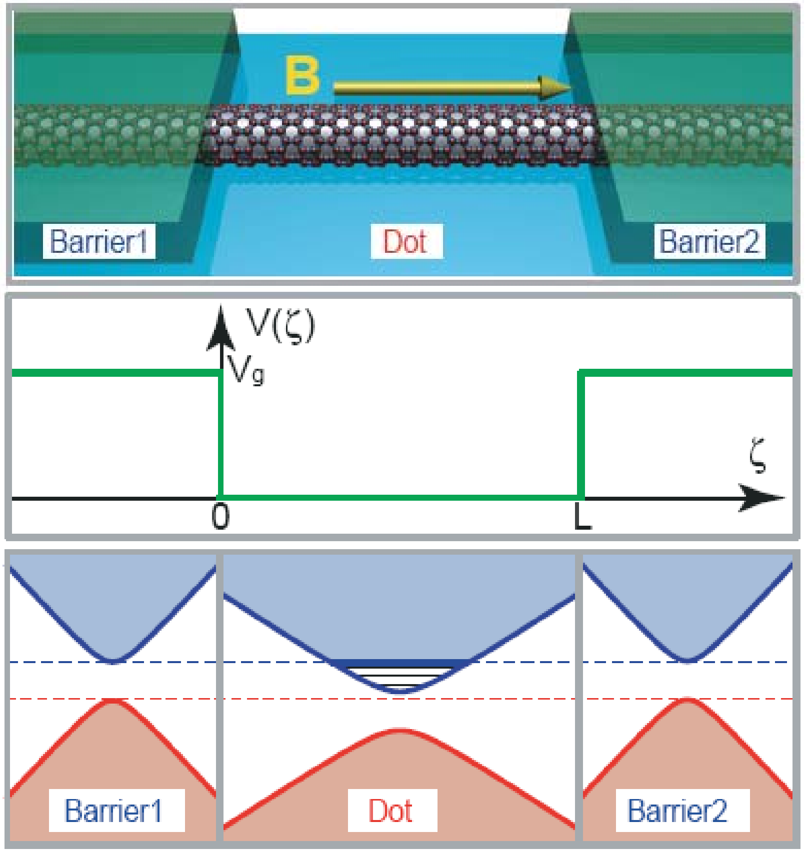
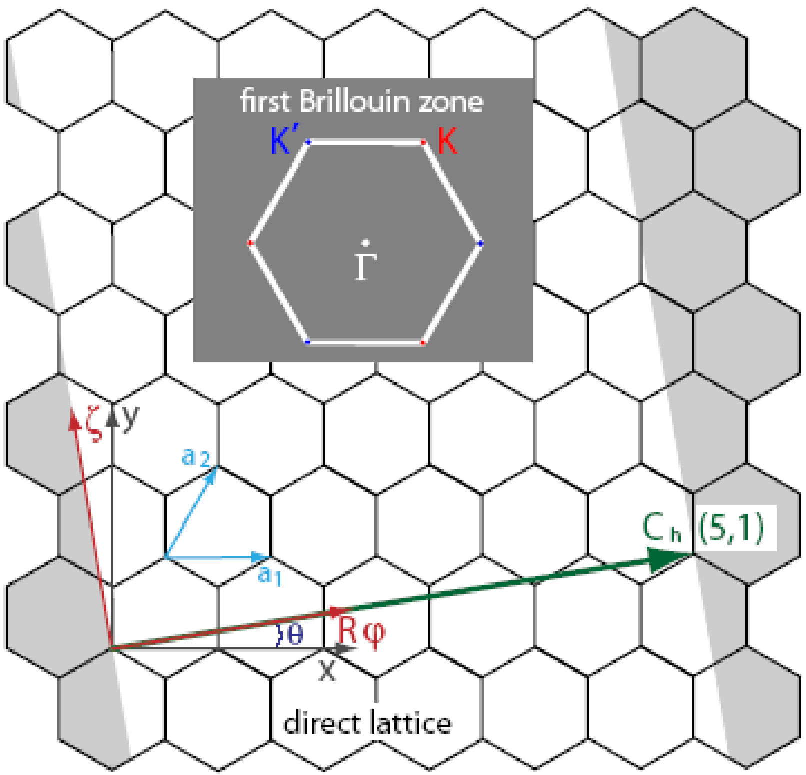
Outline

- **spectrum** of gated SWNTs
- **spin-orbit coupling** in SWNTs
- phonon spectrum and **electron-phonon coupling** in SWNTs
- **spin relaxation times** in SWNT quantum dots

Gated carbon nanotube quantum dot



Hamiltonian of SWNT



$$H_0 = hv\begin{pmatrix} 0 & (\tau_3\kappa - ik)e^{-i\tau_3\theta} \\ (\tau_3\kappa + ik)e^{i\tau_3\theta} & 0 \end{pmatrix}$$

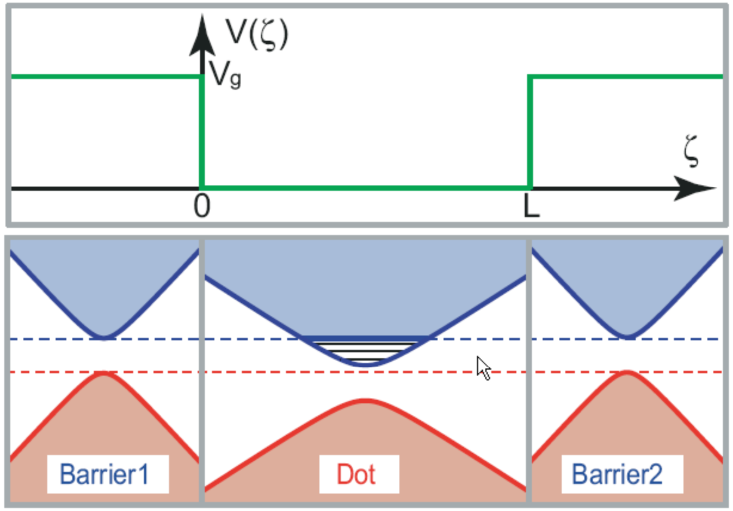
k: longitudinal wave vector
κ: azimuthal wave vector

$$E_{\kappa_m,k,S_\zeta} = \pm\hbar v\sqrt{\kappa_m^2 + k^2} + S_\zeta \hbar\omega_Z$$

$$\Psi^{(')}_{\kappa_m,k,S_\zeta}(\varphi,\zeta) = \Psi^{(')}_{\kappa_m,k}(\varphi,\zeta)\left|S_\zeta\right\rangle$$

Review: *Ando* J. Phys. Soc. Jpn. 2005

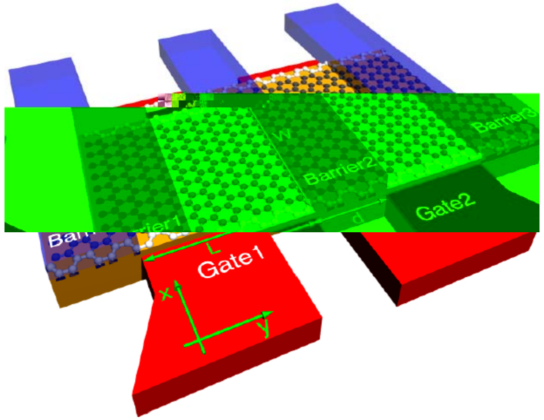
Calculation of bound states



wave function matching ⇒

$$\tan(k_n L) = \frac{(\hbar v)^2 \tilde{k}_n k_n}{E_{\kappa_m . k_n}\left(E_{\kappa_m . k_n} - V_g\right) - (\hbar v)^2 \kappa_m^2}$$

same transcendental equation as for gated graphene nanoribbons



Trauzettel, Bulaev, Loss, Burkard Nature Phys. 2007

Spin-orbit interaction in SWNTs

intrinsic SOI (Dresselhaus-like):

$$H_{SO}^{\text{int}} = \Delta_{\text{int}}\tau_3\sigma_3\left(S_+e^{i\varphi} + S_-e^{-i\varphi}\right)$$

Ando JPSJ 2000; *Kane, Mele* PRL 2005; *Huertas-Hernando et al.* PRB 2006

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extrinsic SOI (Bychkov-Rashba-like):

$$H_{SO}^{E} = \Delta_E \left[i\tau_3 \sigma_1 \left(-S_+ e^{i\varphi} + S_- e^{-i\varphi} \right) + 2\sigma_2 S_\zeta \right]$$

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extrinsic SOI (due to curvature):

$$H_{SO}^{curv} = i \Delta_{curv}^{\perp} \sigma_2 \left(-S_+ e^{i\varphi} + S_- e^{-i\varphi} \right) + \Delta_{curv}^{\parallel} \tau_3 \sigma_1 2 S_\zeta$$

yields spin mixing

yields zero-field splitting

Ando JPSJ 2000; *Kane, Mele* PRL 2005; *Huertas-Hernando et al.* PRB 2006

Hierarchy of energy scales

- For moderate electric fields (E < 0.1 V/nm):

$$\Delta_E < \Delta_{\text{int}} < \Delta_{curv}$$

$\Delta_{\text{int}} \approx 1\mu\text{eV}$

- Curvature-induced SOI is dominant:

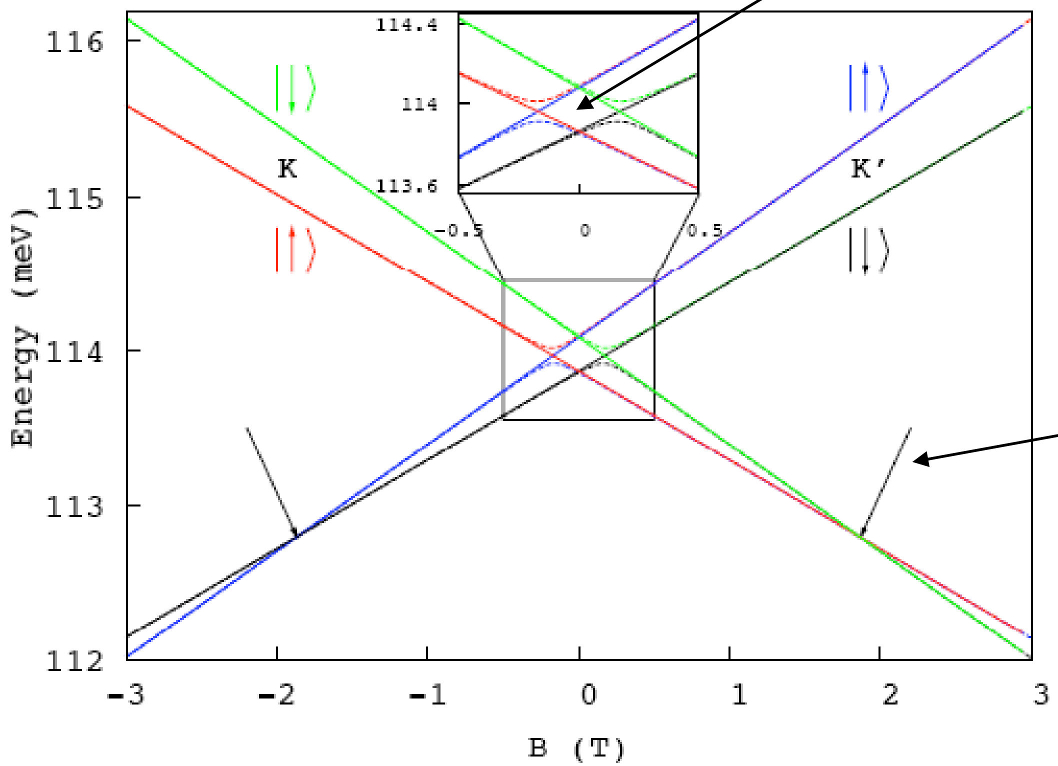
$$\Delta^{\parallel}_{curv} \approx 0.17\text{meV}/\text{R[nm]}$$

$$\Delta^{\perp}_{curv} \approx -0.26\text{meV}/\text{R[nm]}$$

absolute values based on first-principle calculations for graphite

Energy spectrum (including SOI)

ZFS ≈ 0.22 meV for R ≈ 1.6 nm

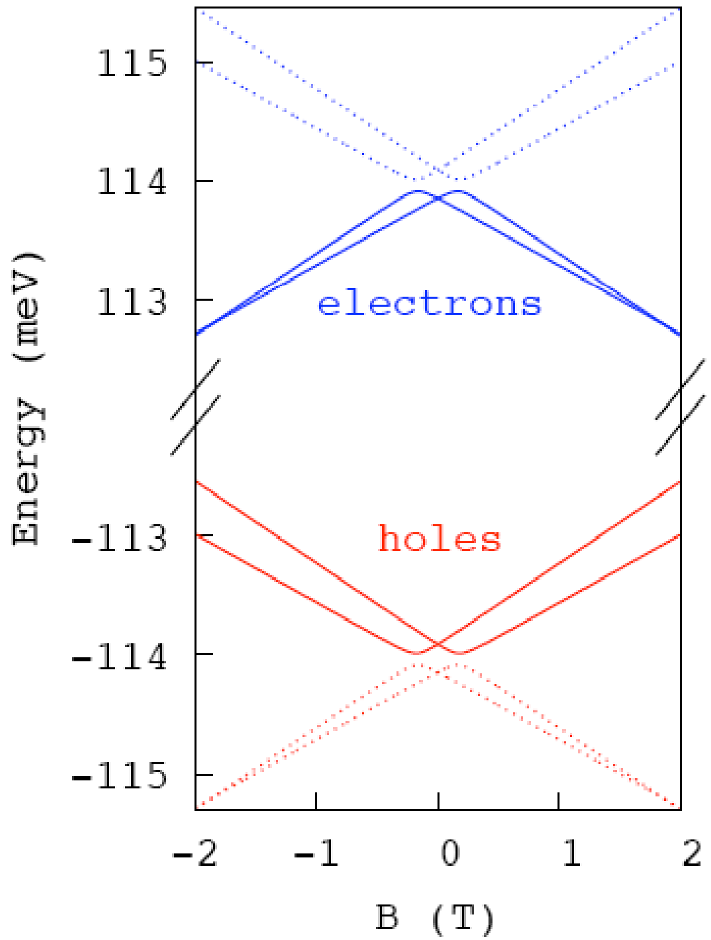


crossings of spin up and spin down

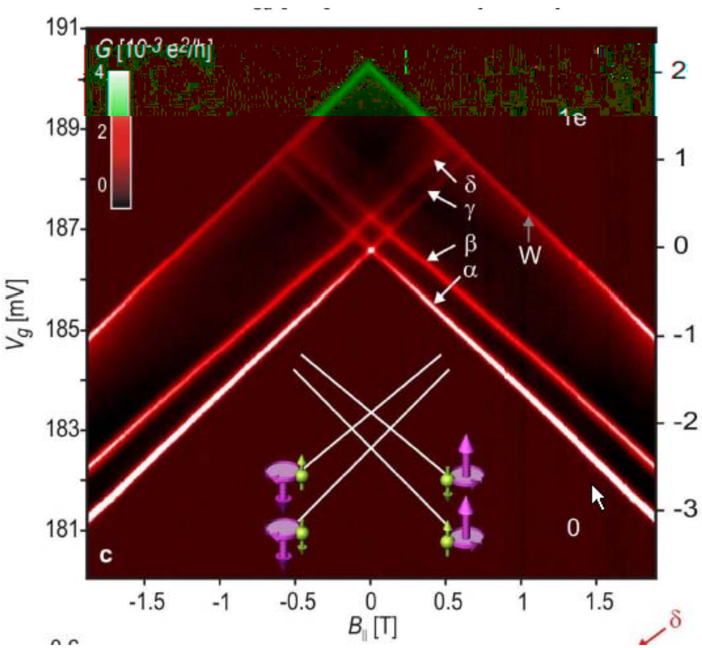
$$H = H_0 + H_Z + H_{SO}^{curv} + H_{K-K'}$$

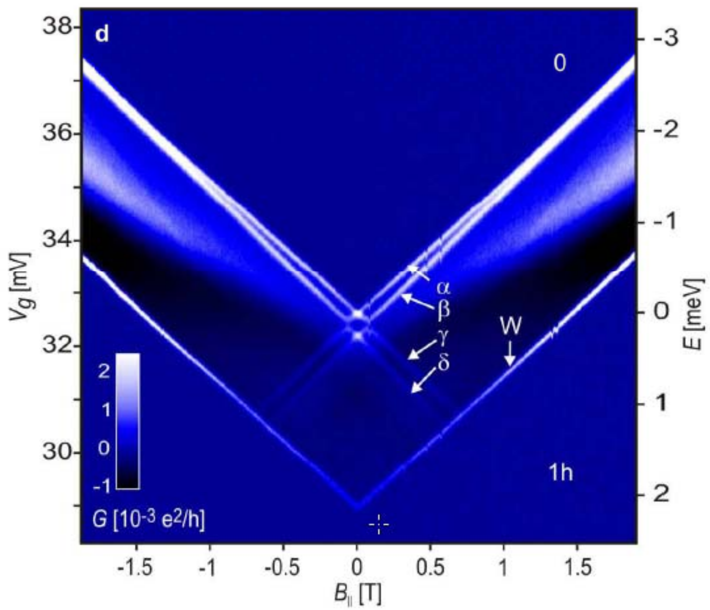
$$H_{K-K'} = \Delta_{K-K'}\tau_1$$

Electron-hole asymmetry (in presence of SOI)



Measurement by Cornell group





electron spectrum

hole spectrum

Kuemmeth, Ilani, Ralph, McEuen, arXiv2008

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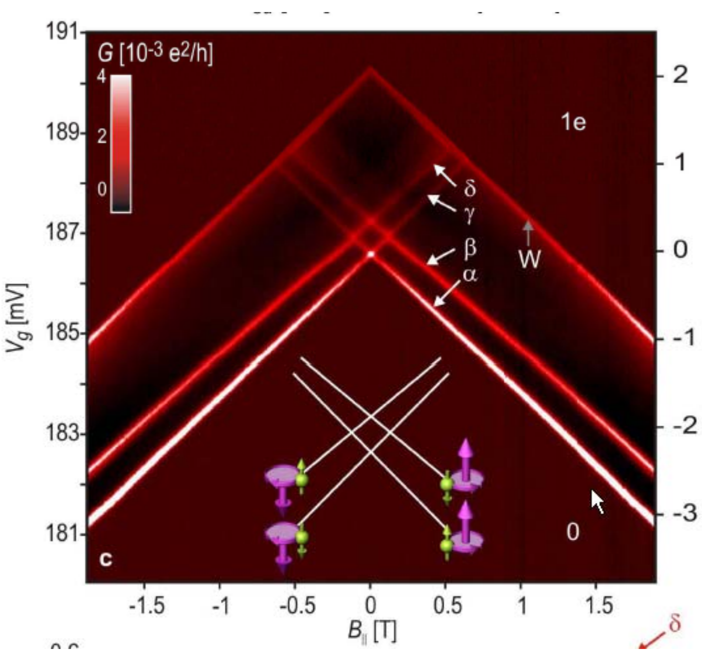
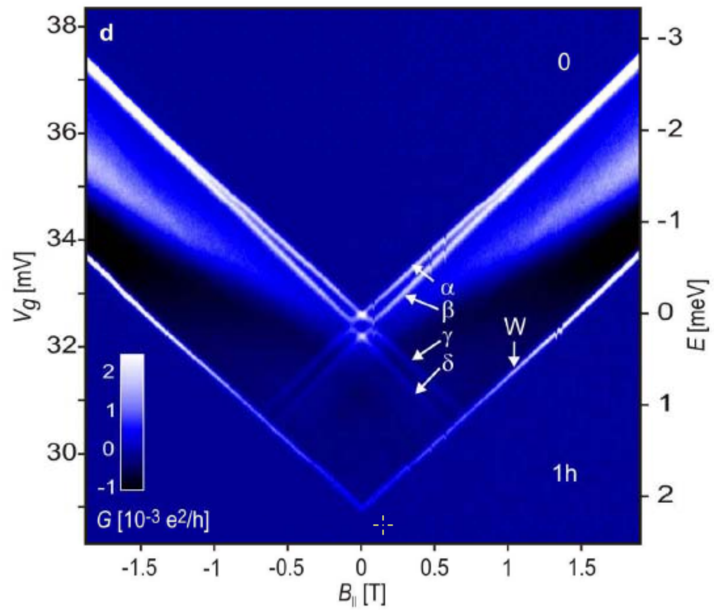




electron spectrum

$2\Delta_{SO} \approx 0.37\text{meV}$

$2\Delta_{K-K'} \approx 65\mu\text{eV}$

hole spectrum

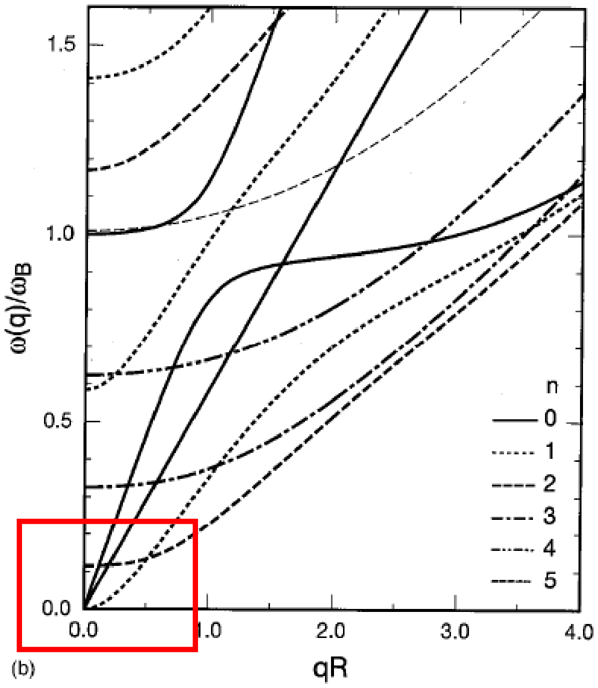
$2\Delta_{SO} \approx 0.21\text{meV}$

$2\Delta_{K-K'} \approx 0.1\text{meV}$

Kuemmeth, Ilani, Ralph, McEuen, arXiv2008

Low-energy phonons in SWNTs

- twisting mode (TM)
- stretching mode (SM)
- bending mode (BM)



Suzuura, Ando PRB 2002

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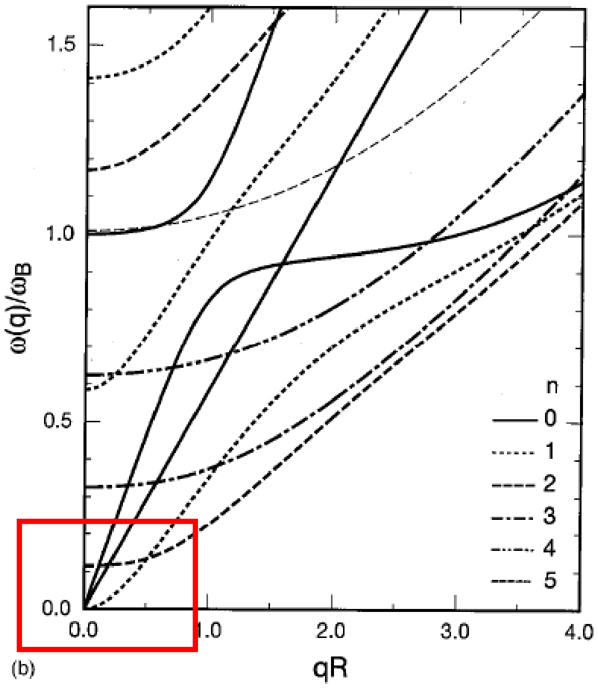

continuum model (with force-constant tensor Λ):

$$\ddot{\mathbf{u}}(\mathbf{r},t) = \Lambda \mathbf{u}(\mathbf{r},t)$$

$$\mathbf{u}(\mathbf{r},t) = (u_\varphi, u_\zeta, u_r)$$

solutions take the form:

$$\mathbf{u}(\mathbf{r},t) = \mathbf{A}\exp[i(m\varphi + q\zeta - \omega t)]$$

only BM has m=1; other modes m=0

Electron-phonon coupling

$$V_{el-ph} = \begin{pmatrix} V_1 & V_2 \\ V_2^* & V_1 \end{pmatrix} + H.c.$$

<u>Deformation potential:</u>

$$V_1 = g_1 \left(u_{\varphi\varphi} + u_{\zeta\zeta} \right)$$

$$u_{\varphi\varphi} = \frac{1}{R}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{R}$$

$$u_{\zeta\zeta} = \frac{\partial u_\zeta}{\partial \zeta}$$

<u>Bond-length change:</u>

$$V_2 = g_2 e^{3i\theta} \left(u_{\varphi\varphi} - u_{\zeta\zeta} + 2i u_{\varphi\zeta} \right)$$

$$2u_{\varphi\zeta} = \frac{\partial u_\varphi}{\partial \zeta} + \frac{1}{R}\frac{\partial u_\zeta}{\partial \varphi}$$

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electron-phonon coupling in SWNTs is quite strong:

$g_1 \approx 30$eV $g_2 \approx 1.5$eV

Lowest energy levels

$$\Psi_{0,0,\pm 1/2} \approx \Psi_{\kappa_0^{\pm},k_0,\pm 1/2} + \sum_{n\neq 0} \lambda_{k_n}^{\pm} \Psi_{\kappa_{\mp 1}^{\mp},k_n,\mp 1/2} + \frac{L}{2\pi} \int_{\pm k_c}^{\infty} dk \lambda_k^{\mp} \Psi_{\kappa_{\mp 1}^{\mp},k,\mp 1/2}$$

level mixing:
discrete spectrum

level mixing:
continuous spectrum

$$\lambda_{k_n}^{\pm} = \pm i \Delta_{curv}^{\perp} \frac{\left\langle \Phi_{\kappa_{\mp 1}^{\mp},k_n}(\zeta) \middle| \sigma_2 \middle| \Phi_{\kappa_0^{\pm},k_0}(\zeta) \right\rangle}{E_{\kappa_{\mp 1}^{\mp},k_n,\mp 1/2} - E_{\kappa_0^{\pm},k_0,\pm 1/2}}$$

$$\kappa_m^{\pm} = \kappa_m \pm \Delta_{curv}^{\parallel} / \hbar v$$

1st order perturbation theory in H_{SO}^{curv}

Spin relaxation time

Using Bloch-Redfield theory:

$$\frac{1}{T_1} = \frac{2\pi}{\hbar}\sum_q (2N_q + 1)\left|M_{\omega_q}\right|^2 \delta\left(\left|E_{\kappa_0^+,k_0,1/2} - E_{\kappa_0^-,k_0,-1/2}\right| - \hbar\omega_q\right)$$

$$\frac{1}{T_1} = \frac{2^{5/4}\pi L}{\hbar^2\sqrt{c_S R\omega_0}}(2N_{\omega_0} + 1)\left|M_{\omega_0}\right|^2$$

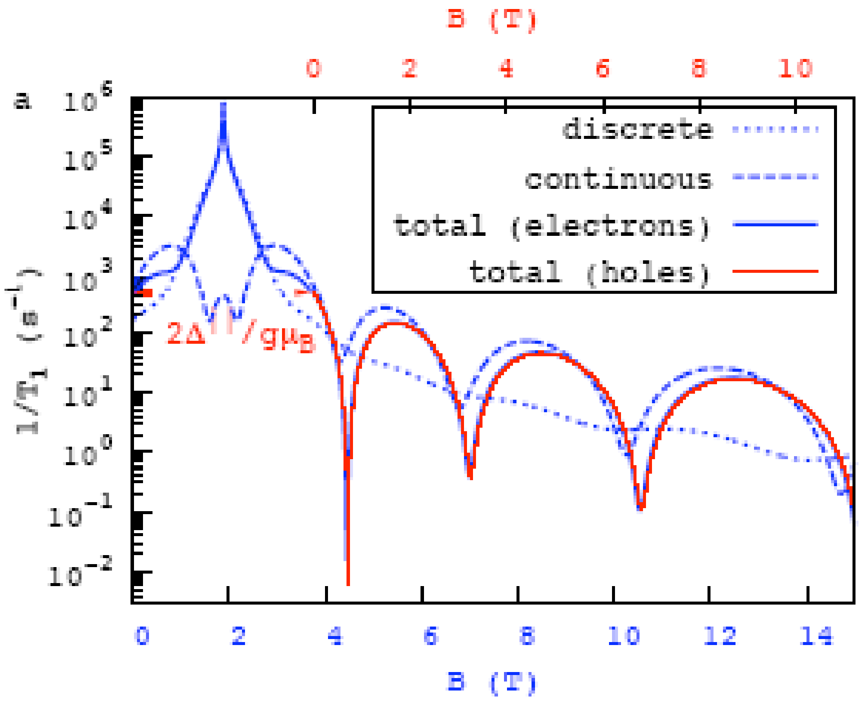
$$\omega_0 = \left|E_{\kappa_0^+,k_0,1/2} - E_{\kappa_0^-,k_0,-1/2}\right|/\hbar \approx \left|\omega_Z - 2\tau_3\Delta_{curv}^{\parallel}\right|$$

Matrix element of spin-flip transitions:

$$M_\omega = \left\langle \Psi_{0,0,-1/2}\middle|V_{el-ph}(\omega)\middle|\Psi_{0,0,1/2}\right\rangle$$

For BM phonons: $T_2 = 2T_1$

Spin relaxation of (40,0) zigzag SWNT

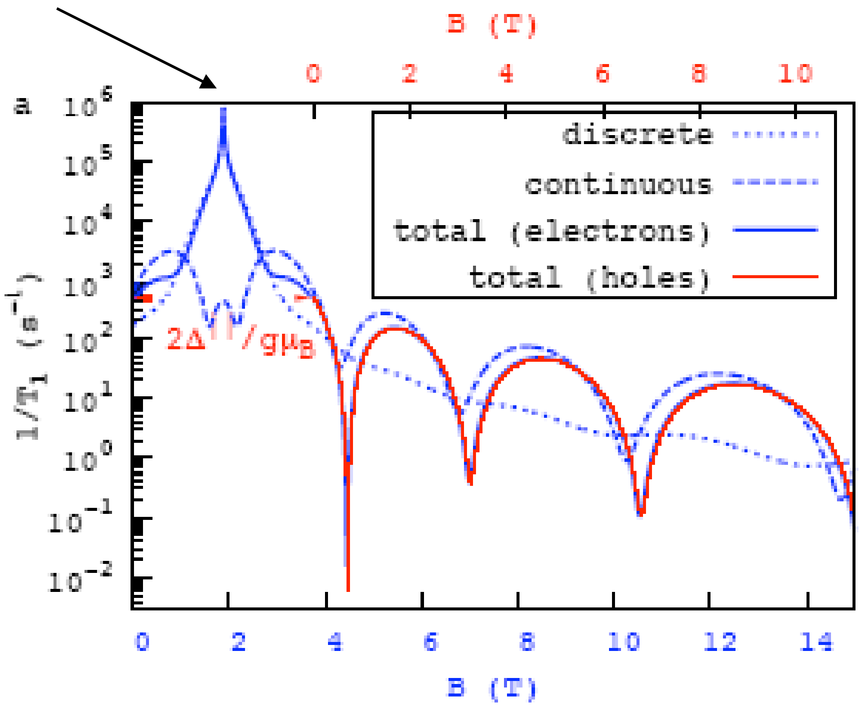


Bulaev, Trauzettel, Loss arXiv 2007

Spin relaxation of (40,0) zigzag SWNT

$$\frac{1}{T_1} \propto \frac{1}{\sqrt{\omega_0}}$$

$$\omega_0 \approx \left|\omega_Z - 2\tau_3\Delta^{\parallel}_{curv}\right|$$



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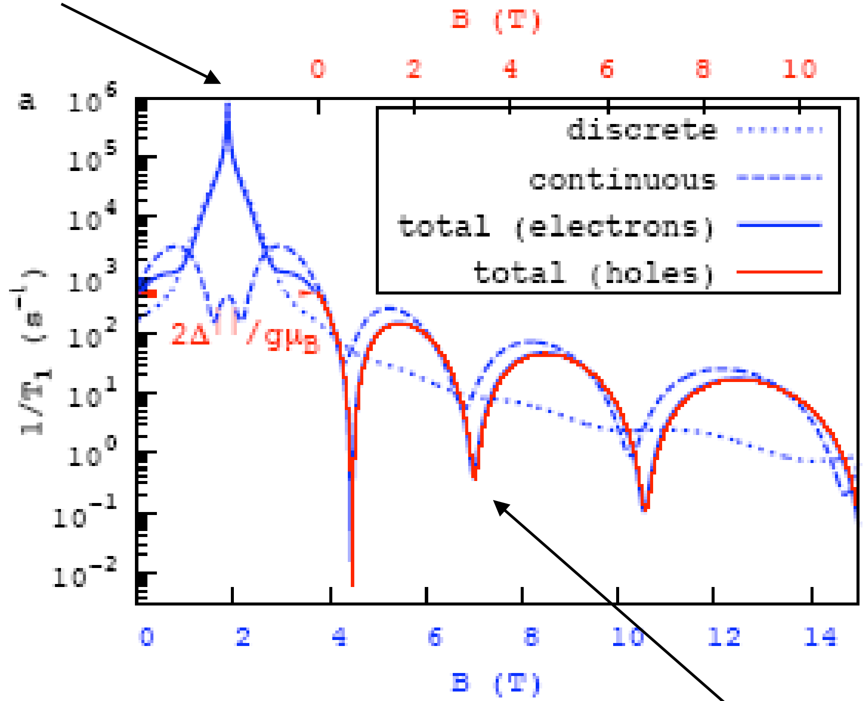
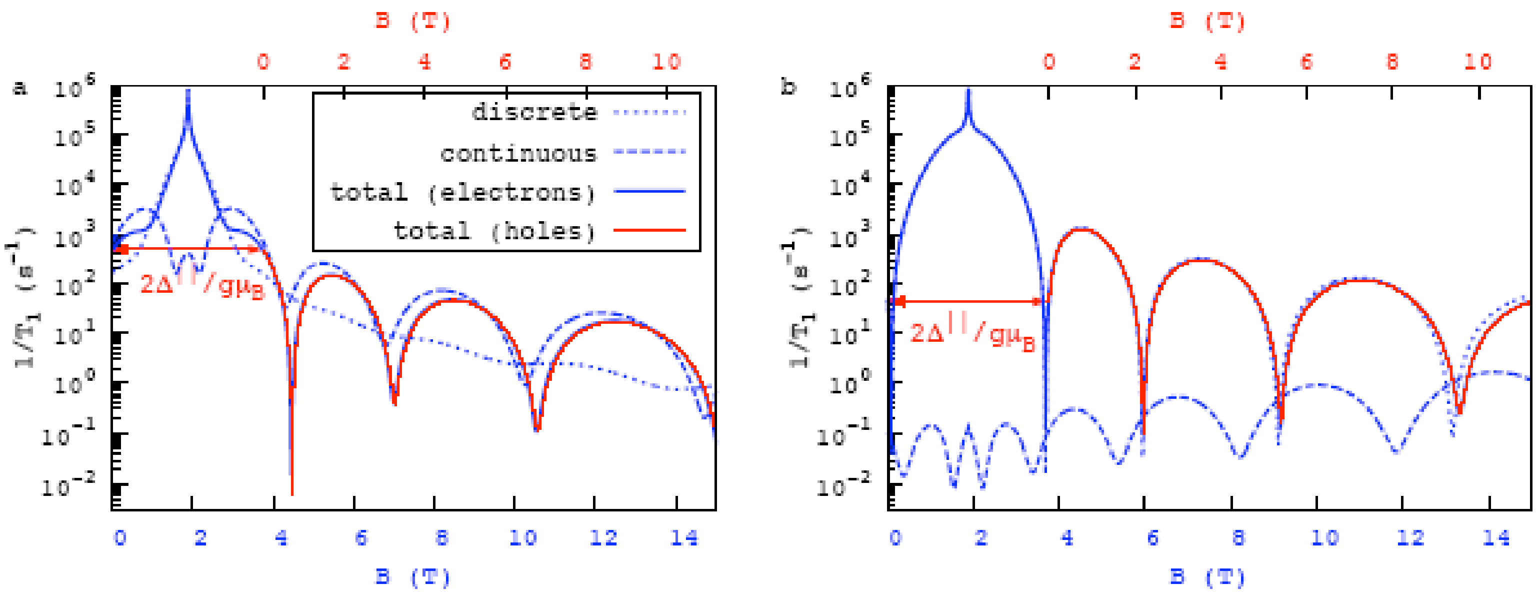


resonances at magic B-field values

Bulaev, Trauzettel, Loss arXiv 2007

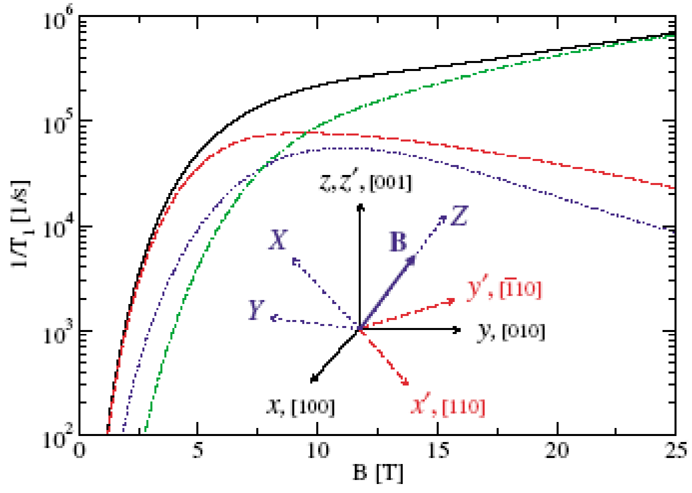
Contributions from discrete vs. continuous part of spectrum



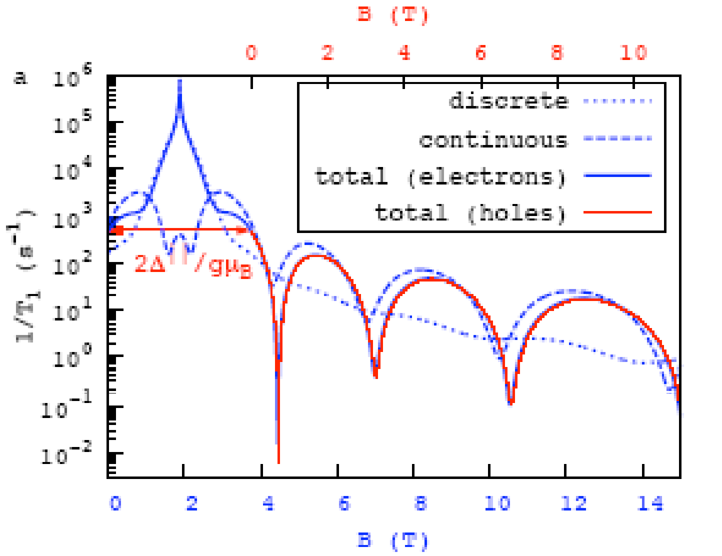
~ 2 quantized levels in each subband

~ 15 quantized levels in each subband

Comparison: GaAs QD vs. SWNT QD

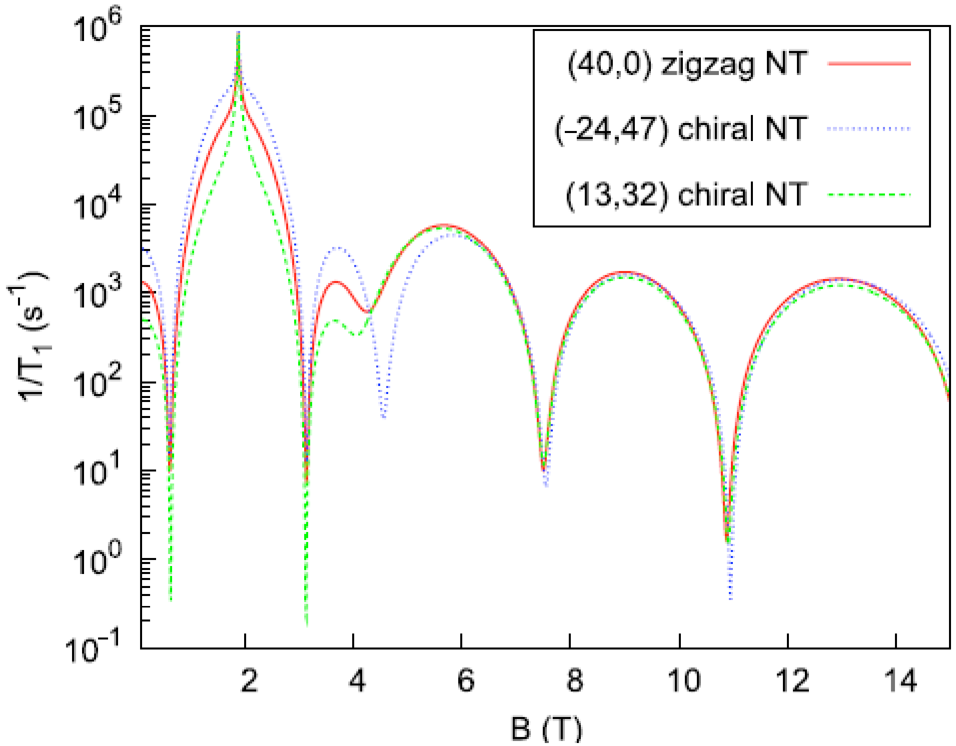


Golovach, Khaetskii, Loss PRL 2004



Bulaev, Trauzettel, Loss arXiv 2007

Chirality dependence of spin relaxation



Conclusions

- SWNTs exhibit **zero-field splitting** due to SOI → all-electrical and phase-coherent control of spin possible

$$H_{SO}^{curv} = i\Delta_{curv}^{\perp}\sigma_2\left(-S_+e^{i\varphi} + S_-e^{-i\varphi}\right) + \Delta_{curv}^{\parallel}\tau_3\sigma_1 2S_\zeta$$

- Interference phenomena result in **ultralong T_1** times (up to 100s) for magic B-field values

- For **small phonon frequencies** ω: $1/\sqrt{\omega}$ spin-phonon noise spectrum