First I would like to discuss general concepts in describing noise and Full Counting Statistics (FCS). I will discuss what is measurable in noise and FCS. Then I will talk about recent development in the description of the Full Counting Statistics (FCS) for electrons in quantum conductor. We (see the list of co-authors after the title) study the FCS for transmission of the finite number of electrons. If just few electrons are send to the scatterer, the exchange effects can be much more prominent. We study the FCS for the transmission of two identical particles with positive or negative symmetry under exchange for the situation where the scattering depends on energy. We find that, besides the expected sensitivity of the noise and higher cumulants, the exchange symmetry has a huge effect on the average transmitted charge. For equal-spin exchange-correlated electrons, the average transmitted charge can be orders of magnitude larger than the corresponding value for independent electrons. Also, we find that the probability of tunneling for two particles can considerably exceed the "classical answer" - product of two probabilities for each particle. The biggest deviation from the classical answer is found in the case when the particles are entangled, when the noise can be super-Poissonian.
In the limit when number of electrons very big, we recovered a set of results previously obtained by using the second-quantization formalism for infinite number of electrons (extended presentation will be given by **Fabian Hassler** at the poster session).

1. **General concepts in describing noise and Full Counting Statistics (FCS)**

2. **What is measurable in noise and FCS**

3. **FCS for transmission of the finite number of electrons, wave packet description**

General concepts in describing noise and Full Counting Statistics (FCS)

• Noise gives more information than average current . Principal e.g. for detecting entanglement via checking violation of the Bell inequality

• FCS is a part of Full Statistical Description, principally allowed by quantum mechanics. The mechanical, deterministic description does not exist

• In addition to making more involved, or complete in some sense measurements, one may study the system in a different conditions, e.g. applying alternating voltage (e.g. photon assisted noise)
Lorentzian pulses of voltage produce exactly one electron excitation above Fermi level
N pulses (even overlapping) produces exactly N electron excitations

• This way one come to a simple controllable system, in particular we studied just two electrons tunneling problem

What is measurable in noise and FCS

- Current operators do not commute at different times
  Current-current correlators at different time are not real value

- Symmetrization – old receipt from e.g. Landau Lifshitz

$$S(\omega) = \int \mathrm{d}t \, \exp(\mathrm{i}\omega t) \left\langle \frac{1}{2} \{ I(0) I(t) + I(t) I(0) \} \right\rangle .$$

- Imply that zero-point fluctuations in current are measurable

$$S(\Omega) = 2G\hbar\Omega \left[\frac{1}{2} + \frac{1}{\exp(\hbar\Omega / k_{\mathrm{B}} T) - 1} \right]$$

What is measurable in noise and FCS

- **Non-symmetrical spectral density – Lesovik, Loosen 1997**

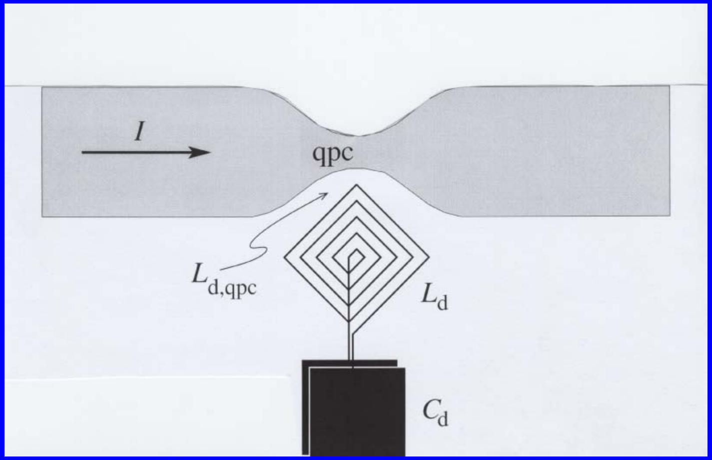
$$S_{\text{meas}} = K\{S_+(\Omega) + N_\Omega[S_+(\Omega) - S_-(\Omega)]\},$$

with the definitions

$$S_+(\Omega) = \int dt \langle I(0)I(t)\rangle \exp(i\Omega t),$$

and

$$S_-(\Omega) = \int dt \langle I(t)I(0)\rangle \exp(i\Omega t).$$



Non-symmetrical spectral density – Lesovik, Loosen 1997; Gavish, Levinson, Imry 2000; Aguado, Kouwenhoven 2000

Confirmed in several experiments – R. Deblock, E. Onac, L. Gurevich, L.P. Kouwenhoven (2003); P.M. Billangeon et al (2006); E. Zakka-Bajjani et al (2008)

$$S_+(\Omega) = \frac{2e^2}{h} D(1-D)(eV - \hbar\Omega)$$

if $\hbar\Omega < eV$ and $S_+(\Omega) = 0$ otherwise.

What is measurable in noise and FCS

- Imply that zero-point fluctuations in current are measurable

$$S(\Omega) = 2G\hbar\Omega\left[\frac{1}{2} + \frac{1}{\exp(\hbar\Omega/k_{\mathrm{B}}T) - 1}\right]$$

Measurable, theory - Lesovik (UFN 1998), no experiment

$$\langle x(0)x(t)\rangle = \lambda^2\left(\frac{2}{\gamma}\right)^4\left\{\langle\hat{I}(0)\hat{I}(t)\rangle_{\mathrm{S}} - \mathrm{i}\langle\hat{I}(0)\hat{I}(t)\rangle_{\mathrm{A}}\frac{8T_x}{\hbar\gamma}\right\}.$$

$$\ddot{\phi} = -\Omega^2\phi - \gamma\dot{\phi} + \lambda I(t)\,.$$

What is measurable in noise and FCS

- In FCS definition for symmetrized cumulants in charge

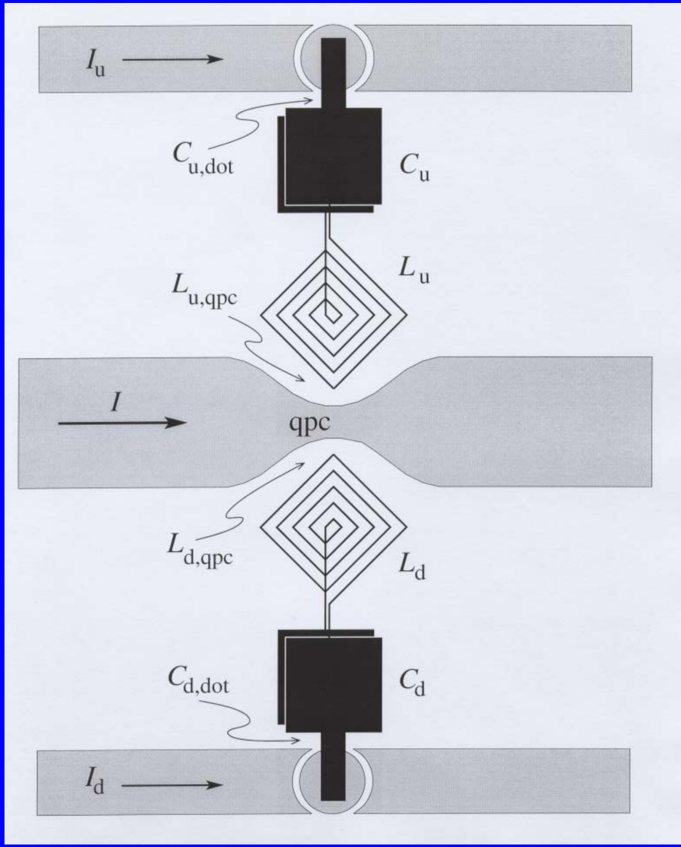
$$\chi(\lambda) = \left\langle \exp\left\{ i\lambda \int_0^t \hat{I}\, dt' \right\} \right\rangle$$

Levitov Lesovik 1992, not measurable, but!

This calculation gives as a byproduct correct values for zero-frequency current-current correlators of all orders (Lesovik Chtchelkatchev, 2003)

$$\langle\langle \hat{Q}(t_0)^n \rangle\rangle = t_0 \langle\langle \hat{I}_0^n \rangle\rangle,$$

Zero-frequency current-current correlators of all orders can be measured – for 4-order correllator the set-up discused by A.Lebedev, G.Blatter, G.Lesovik (unpublished 2008)



Spin-detector Levitov Lesovik 1994 led to different definition for the high-order cumulants of the transmitted charge which includes the time ordering

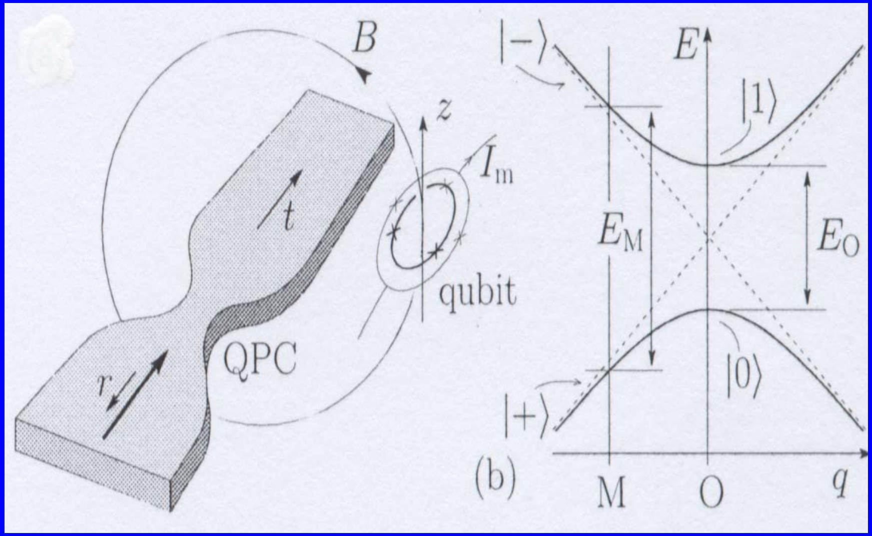
$$\chi(\lambda) = \left\langle \tilde{T}\exp\left(\frac{i\lambda}{2}\int_0^t \hat{I}\,dt'\right) T\exp\left(\frac{i\lambda}{2}\int_0^t \hat{I}\,dt''\right)\right\rangle,$$

Lesovik Hassler Blatter (2006): the Levitov-Lesovik definition (1994) for the characteristics function is equivalent to the Fidelity for the wave function propagating in the spin-detector field

$$\begin{aligned}\chi_{\mathrm{fid}}(\lambda, t) &= \langle \Psi_\lambda(t) | \Psi(t) \rangle \\ &= \langle \Psi | e^{i(H_{\mathrm{sys}} + \lambda h)t/\hbar} e^{-iH_{\mathrm{sys}} t/\hbar} | \Psi \rangle\end{aligned}$$

$$\rho^{\mathrm{S}}_{\sigma_z', \sigma_z}(t) = \rho^{\mathrm{S}}_{\sigma_z', \sigma_z}(0) \times \mathrm{Tr}\left[\rho^{\mathrm{sys}}(0)\, e^{i(H_{\mathrm{sys}} + \lambda h \sigma_z)t/\hbar} e^{-i(H_{\mathrm{sys}} + \lambda h \sigma_z')t/\hbar}\right]$$

$$\chi_1(\lambda) = \int dx\, \psi^-_{\mathrm{out}}(x;t)^* \, \psi^+_{\mathrm{out}}(x;t)$$



FCS for transmission of the finite number of electrons, wave packet description

• Th.Martin, R.Landauer, PRB 1992

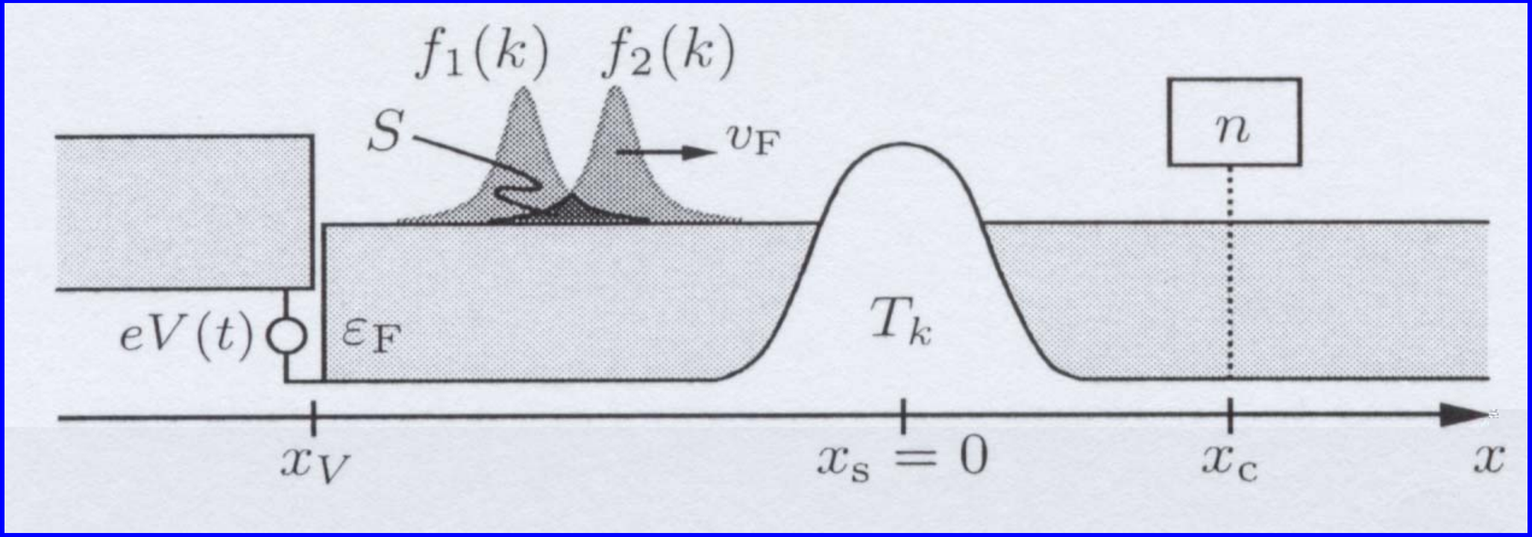
Individual electrons

• Excitation by Lorentzian voltage pulses suggested by Ivanov, Lee, Levitov 1995

• Experiment - e.g. «An On-Demand Single Electron Source» Feve et al (2007)

Two electrons

• Exchange effects Hassler, Lesovik, Blatter 2007



The properly (anti-)symmetrized two-particle wave functions assume the form $\psi_{\mathrm{x},\pm}(x_1, x_2; t) \propto \psi_{\mathrm{x},1}(x_1; t)\psi_{\mathrm{x},2}(x_2; t) \pm (x_1 \leftrightarrow x_2)$ with x = in, out. The transport statistics is described by the generating func-

Fourier transform $P_n = \int (d\lambda/2\pi)\chi(\lambda)e^{-in\lambda}$ yields the probability P_n for transmitting n particles. The '±' signs in Eq. (3) refer to the symmetry under exchange of the

the exchange symmetry of the particles.

$$P_{0,\pm} = \frac{(1 - \langle T \rangle)^2 \pm |S - \langle 1|T|2 \rangle|^2}{1 \pm |S|^2},$$

$$P_{1,\pm} = 2\frac{\langle T \rangle(1 - \langle T \rangle) \pm \left[\mathrm{Re}(\langle 1|T|2 \rangle S^*) - |\langle 1|T|2 \rangle|^2\right]}{1 \pm |S|^2},$$

$$P_{2,\pm} = \frac{\langle T \rangle^2 \pm |\langle 1|T|2 \rangle|^2}{1 \pm |S|^2};$$

Quantum point contact weakly open channel

$$\langle Q/e \rangle_-^{\mathrm{qpc}} = 2\langle T^{\mathrm{qpc}} \rangle (1 + 2\xi^2 k_0^2).$$

Diagonalization for Slater determinant

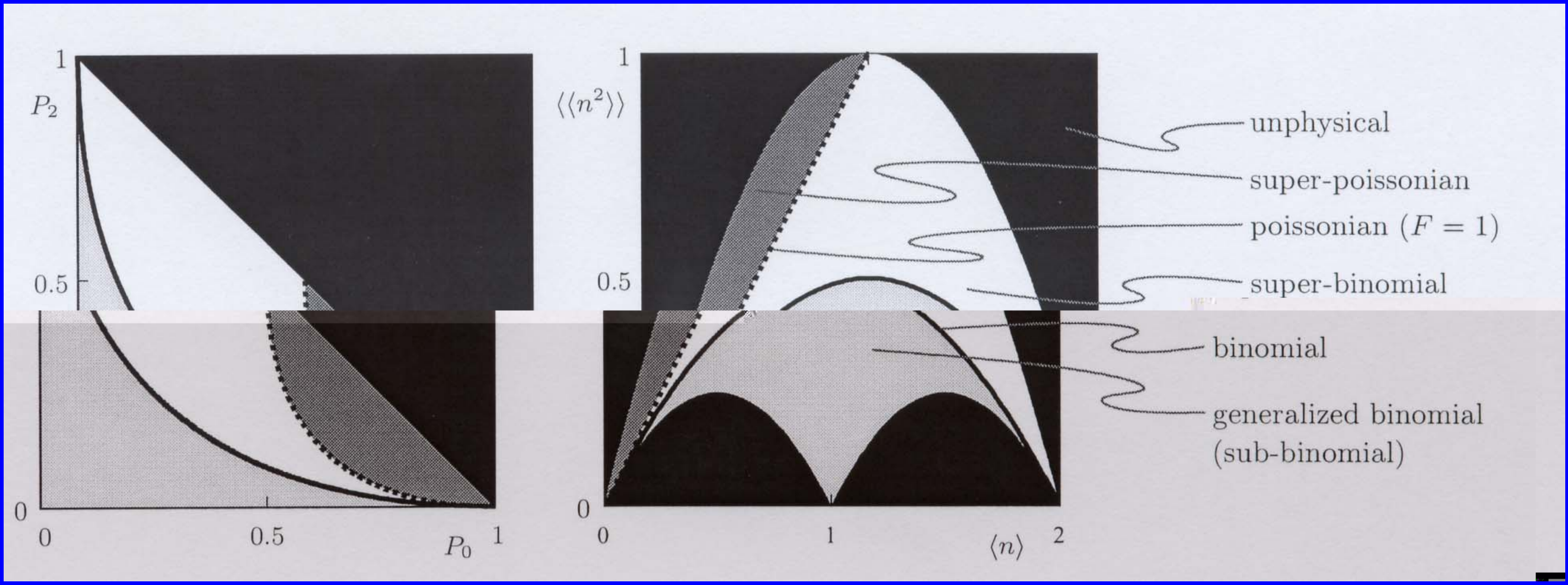
$$\chi_{1,\pm 1}(\lambda) = (1-\tau_1+\tau_1 e^{i\lambda})(1-\tau_2+\tau_2 e^{i\lambda})$$

$$= \frac{(1-\mathsf{T}^f_{11}+\mathsf{T}^f_{11}e^{i\lambda})(1-\mathsf{T}^f_{22}+\mathsf{T}^f_{22}e^{i\lambda})}{1-|S|^2}$$

$$- \frac{(S-\mathsf{T}^f_{21}+\mathsf{T}^f_{21}e^{i\lambda})(S^*-\mathsf{T}^f_{12}+\mathsf{T}^f_{12}e^{i\lambda})}{1-|S|^2}$$

$$\tau_{1,2} = \frac{\alpha \mp \sqrt{\alpha^2 - \det \mathsf{T}^f \det \mathsf{S}^f}}{\det \mathsf{S}^f},$$

where the parameter $2\alpha = \mathsf{S}^f_{22}\mathsf{T}^f_{11} + \mathsf{S}^f_{11}\mathsf{T}^f_{22} - 2\mathrm{Re}(\mathsf{S}^f_{12}\mathsf{T}^f_{[illegible]})$



$$P_0 \le 1 - T_{\max}, \quad P_1 \ge T_{\max} - T_{\min}, \quad P_2 \le T_{\min}.$$

$$F = \frac{\langle\langle n^2\rangle\rangle}{\langle n\rangle} \le 1 - \langle n\rangle/N \le 1$$

Entangled states

Sukhorukov, Loss, Burkard 1999
Taddei, Fazio 2001

Our recent work of 2008:

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Extended presentation will be given by Fabian Hassler at the poster session