1D anyons: statistical mechanics and Coulomb blockade

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D.V.A. and J.A. Nesteroff, Physica E, **40**, 58 (2007);
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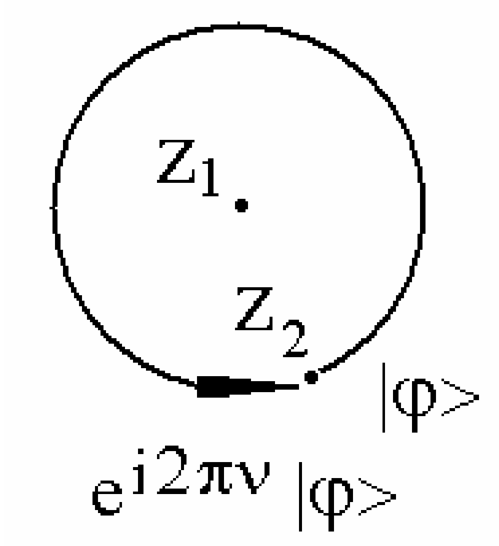
Outline

1. Introduction.
2. 1D anyons: wavefunctions and correlators
3. Coulomb blockade of anyons in quantum antidots
   (a) Quasiparticles on antidots: hard-core anyons.
   (b) Edges as reservoirs:
      - Luttinger-liquid correlations in the tunneling rates;
      - decoherence/dissipation due to edge resistance.
   (c)Triple-antidot: quasiparticle statistics in resonant conductance.

Introduction: anyons

In 2D, and for hard-core particles, one can unambiguously define the notion of braiding of the particle trajectories, i.e. states with different number of times the trajectory of one particle encircles another particle are not identical. This makes it possible to consider "anyonic" particles with fractional exchange statistics.



By contrast, in 3D, one can define only permutations (no braiding) of point-like particles leading to only integer statistics, i.e. $\nu = 0,1$ for bosons and fermions, respectively.

$$\psi(z_2, z_1) = \pm\psi(z_1, z_2).$$

J.M. Leinaas and J. Myrheim, Nuovo Cimento B **37**, 1 (1977).
F. Wilczek, Phys. Rev. Lett. **49**, 957 (1982).

Fractional statistics of FQHE quasiparticles

Geometric phase in cyclic adiabatic evolution

$$H[z(t)]\psi(t) = E(t)\psi(t), \quad \psi(T) = \psi(0)e^{-i\phi+i\gamma},$$

$$\phi = -(i/\hbar)\int_0^T E(t)dt, \quad \gamma = i\oint\langle\psi\,|\nabla_z\psi\rangle dz.$$

For a quasihole rotating around

- the origin:

$$\gamma = i\oint dz_0\langle\psi(\{z_j\},z_0)\,|\nabla_{z_0}\psi(\{z_j\},z_0)\rangle = i\oint dz_0\int ds\rho(z)\nabla_{z_0}\ln(z-z_0) =$$

$$= -2\pi\nu\Phi/\Phi_0'.$$

This is manifestation of the fractional quasihole charge νe.

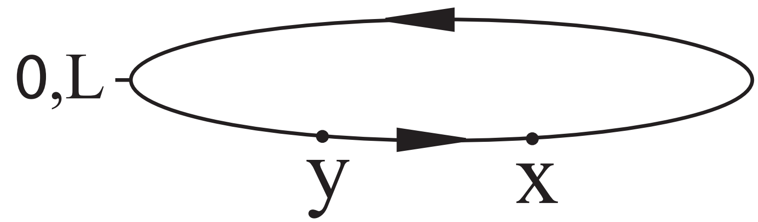
- another quasihole : $\gamma = 2\pi\nu.$ (fractional statistics)

D. Arovas *et al.*, PRL **53**, 722 (1984).

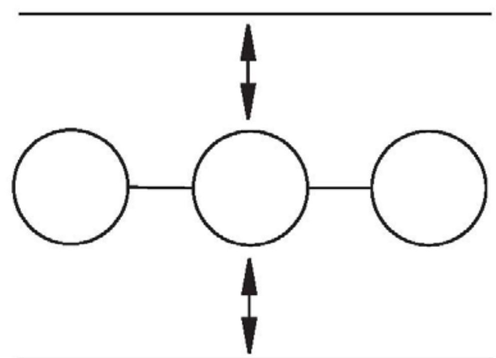
1D anyons – why statistics can be relevant

In the zero-th approximation, hard-core particles do not have statistics in one dimension. How the statistics can still play a role?

- For particles on a circle (quasi-periodic boundary conditions), braiding is possible even in a strictly 1D geometry.



- In the case of tunneling into the system, superposition of different orderings is possible even with closed boundary conditions, and no explicit exchanges.

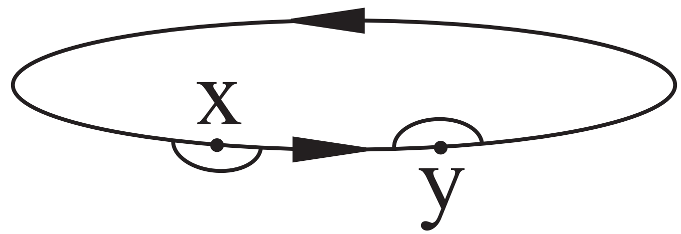


1D anyons - elementary quantum mechanics

- Wavefunctions:

$$\chi(\cdots,x,y,\cdots) = e^{i\pi\nu\epsilon(x-y)}\chi(\cdots,y,x,\cdots),$$
$$\epsilon(x) = \text{sign}(x), \quad \epsilon(0) = 0.$$

One needs to choose a sign of statistical phase for each pair of particles. Natural choice: sign(i-j), for particle numbering z_j.



- Boundary conditions:

$$\begin{aligned}
\chi(0,z_2,\cdots,z_N) &= e^{-i\phi} \quad \chi(L,z_2,\cdots,z_N)\,, \\
\chi(z_1,0,\cdots,z_N) &= e^{i[2\pi\nu-\phi]}\chi(z_1,L,\cdots,z_N)\,, \\
&\vdots \\
\chi(z_1,z_2,\cdots,0) &= e^{i[2\pi(N-1)\nu-\phi]}\chi(z_1,z_2\cdots,L)\,.
\end{aligned}$$

- Shift of particle momenta λ_j:

$$\boxed{\phi \to \overline{\phi} = \phi - \pi\nu(N-1), \qquad e^{i\lambda_j L} = (-1)^{N-1}e^{i\overline{\phi}}}$$

1D anyons - field correlators

Anyons with delta-function repulsion

$$H = \int dx \ \left[-\Psi^\dagger(x)\partial_x^2\Psi(x) + c\Psi^\dagger(x)\Psi^\dagger(x)\Psi(x)\Psi(x) - \mu\Psi^\dagger(x)\Psi(x) \right]$$

$$\Psi(x_1)\Psi^\dagger(x_2) = e^{-i\pi\nu\epsilon(x_1-x_2)}\Psi^\dagger(x_2)\Psi(x_1) + \delta(x_1 - x_2)$$

$$\Psi(x_1)\Psi(x_2) = e^{i\pi\nu\epsilon(x_1-x_2)}\Psi(x_2)\Psi(x_1)$$

Large-distance asymptotic of the field correlator

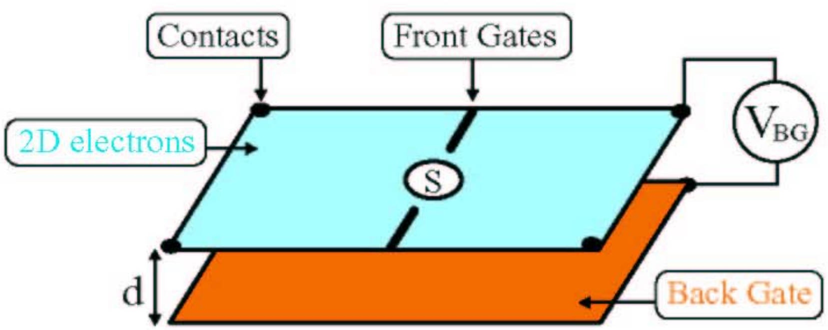
$$\langle \Psi(x)\Psi^\dagger(0) \rangle \simeq e^{i\nu k_F x} / |x|^{(g\nu^2 + g^{-1})/2}$$

reproduces edge-state quasiparticles for $g=1/\nu$.

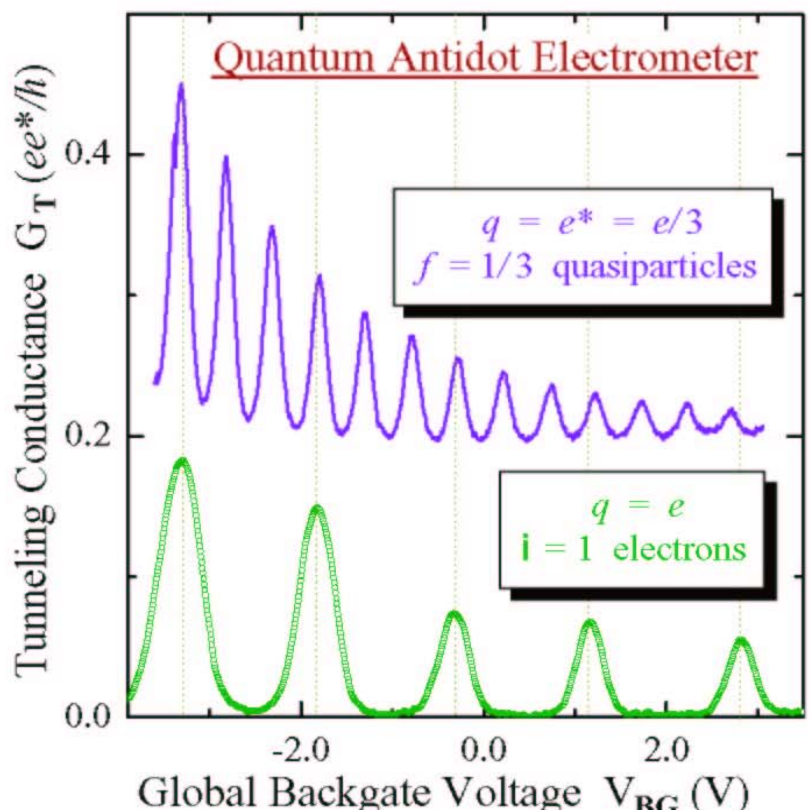
R. Santachiara et al, 2006.
P. Calabrese and M. Mintchev, 2007.

Fractional Charge of Laughlin Quasiparticles

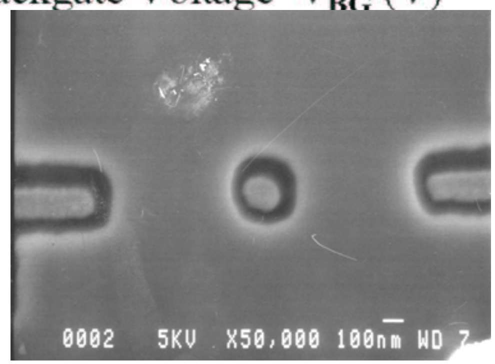
- back gate: $E_{\perp} = \dfrac{V_{BG}}{d}$
- induced surface charge: $\delta\sigma = \varepsilon\varepsilon_0 E_{\perp}$
- one particle per ΔV_{BG}
- particle charge : $q = \varepsilon\varepsilon_0 S \, \Delta E_{\perp}$



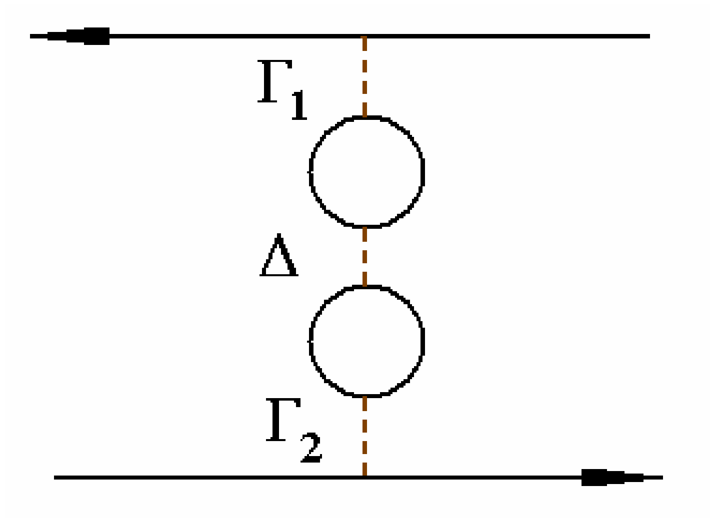
$$q = \frac{\varepsilon\varepsilon_0 \Phi_0'}{d} \frac{\Delta V_{BG}}{\Delta B}.$$



V.J. Goldman and B. Su, 1995



Multi-antidot systems



Antidots

Large number, n>>1, of quasiholes:

$$\psi^{(n)}(\{z_j\}) = \prod_j (z_j - \zeta)^n \psi_m(\{z_j\}), \quad \nu = 1/(2m+1), \quad R^{(n)} \cong \sqrt{2n}\ell,$$

$$\delta R = R^{(n+1)} - R^{(n)} \approx \ell / \sqrt{n} << \ell.$$

But not too large:

$$\Delta^* \approx \hbar u / R > T, \Delta, U \ldots$$

Then the antidot quasiparticle operators

$$\psi(z) = (1/2\pi\alpha)^{1/2}\,\xi\, e^{i\sqrt{\nu}\phi(z)}$$

are reduced to Klein factors:

$$e^{i\sqrt{\nu}\phi(z)} \Rightarrow \langle 0|e^{i\sqrt{\nu}\phi(z)}|0\rangle; \quad \psi = (1/\pi R)^{1/2}\,\xi\,.$$

At most one quasiparticle per antidot in a given energy range + fractional exchange statistics => hardcore anyons.

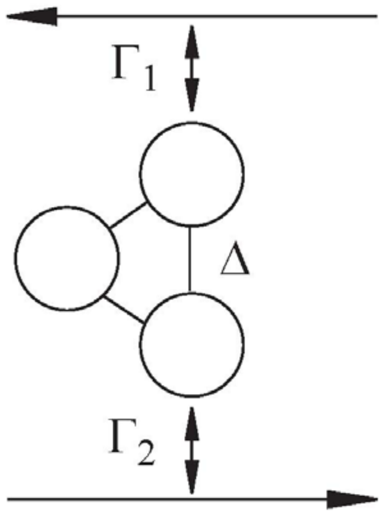
For quasi-1D patterns of tunnel coupling of the antidots, they are described by the Wigner-Jordan transformation:

$$\xi_x = e^{i\pi(\nu-1)\Sigma_{y<x} n_y}\, c_x\,,$$

so that

$$\xi_x \xi_y = \xi_y \xi_x e^{i\pi\nu\,\mathrm{sgn}(x-y)}, \quad \xi_x^2 = 0\,.$$

Hard-core 1D anyons on the antidots: wavefunctions and tunneling



$$H=\sum_x[\varepsilon_x n_x-(\Delta_x\xi_{x+1}^{+}\xi_x+h.c.)]+\sum_{x<y}U_{x,y}n_xn_y\,.$$

$$U_{x,y}=U:\quad \sum_{x<y}U_{x,y}n_xn_y=Un(n-1)/2\,.$$

Two anyons:

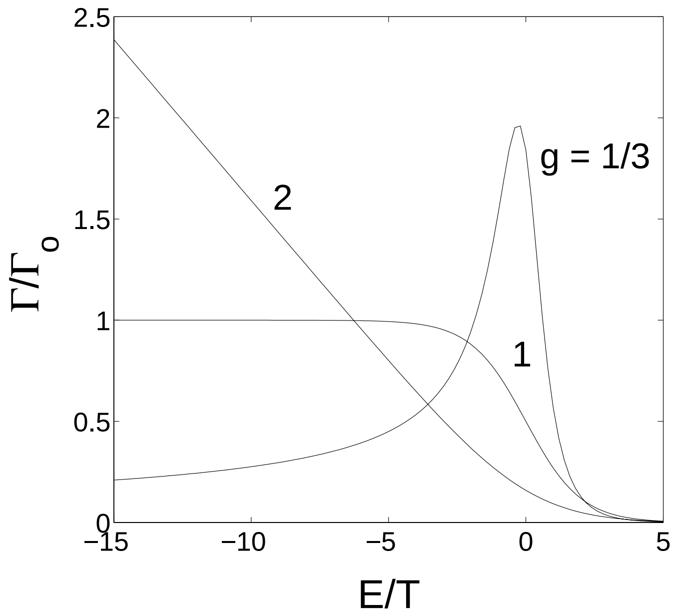
$$\psi(x,y)=(1/\sqrt{2})e^{i\pi(1-\nu)\mathrm{sgn}(x-y)/2}\det\begin{pmatrix}\phi_q(x) & \phi_q(y)\\ \phi_p(x) & \phi_p(y)\end{pmatrix}.$$

Edges as reservoirs

- Tunneling rates reflect Luttinger-liquid correlations in the edges:

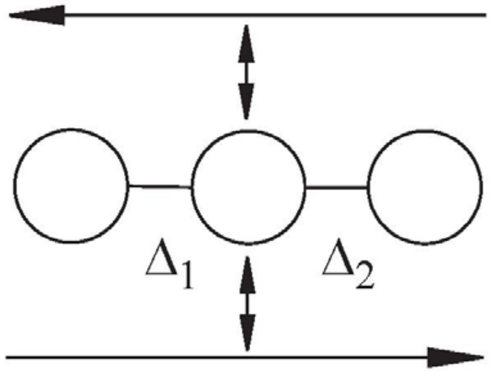
$$\Gamma_j(E) = \gamma_j f_g(E)\,, \quad \gamma_j = 2\pi\,|T_j|^2/\omega_c\,,$$

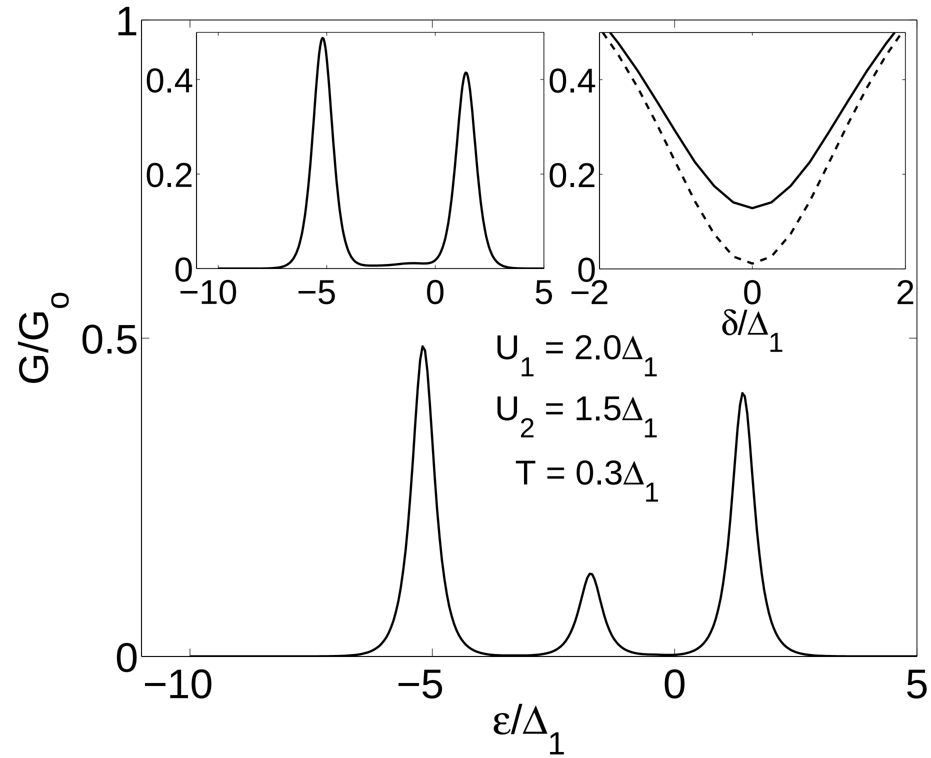
$$f_g(E) = \frac{1}{2\pi\Gamma(g)}\left(\frac{2\pi T}{\omega_c}\right)^{\nu-1}\left|\Gamma\left(\frac{g}{2}+\frac{iE}{2\pi T}\right)\right|^2 e^{-E/2T}$$



- Edge resistance can not be minimized, producing an unavoidable decoherence mechanisms for quasiparticles on the antidots.

Line of antidots: exchange statistics in dc conductance





Resonant conductance peaks, T<Δ,U:

$$G = \frac{(e\nu)^2}{T}\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\frac{a_n f_\nu(E_{n+1}-E_n)}{1+\exp[(E_{n+1}-E_n)/T]}.$$

The amplitude of the middle peak is sensitive to the exchange statistics

$$a_1 = \frac{\Delta_1^2\Delta_2^2}{(\omega_e+\lambda)\omega_e(\omega_h-\lambda)\omega_h}\cos^2(\pi\nu/2).$$

Conclusions

- ✓ One can built elementary quantum mechanics of 1D anyons
- ✓ $$\langle \Psi(x)\Psi^\dagger(0)\rangle \simeq e^{i\nu k_F x}/|x|^{(g\nu^2+g^{-1})/2}$$
- ✓ FQHE quasiparticles on antidots should be treated as the hard-core anyons;
- ✓ Fractional quasiparticle statsictics should manifest itself directly in the dc conductance of a triple-antidot system.