Current noise analysis for SU(4) Kondo screening in QDs

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ENS mesoscopic group

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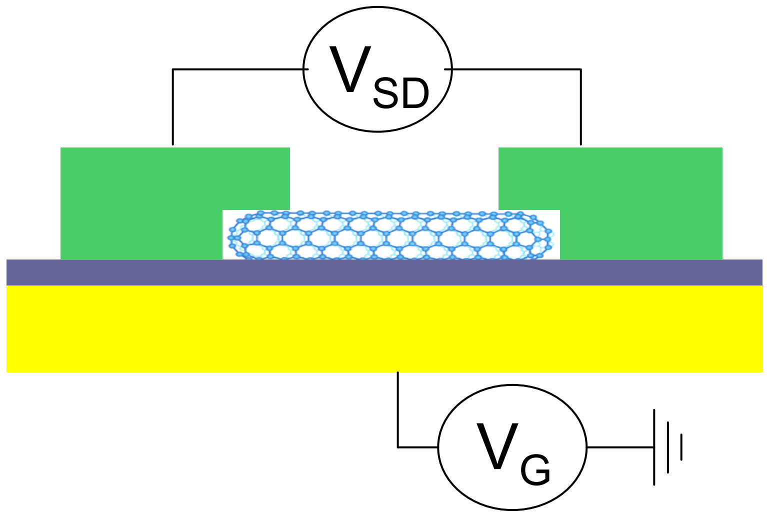
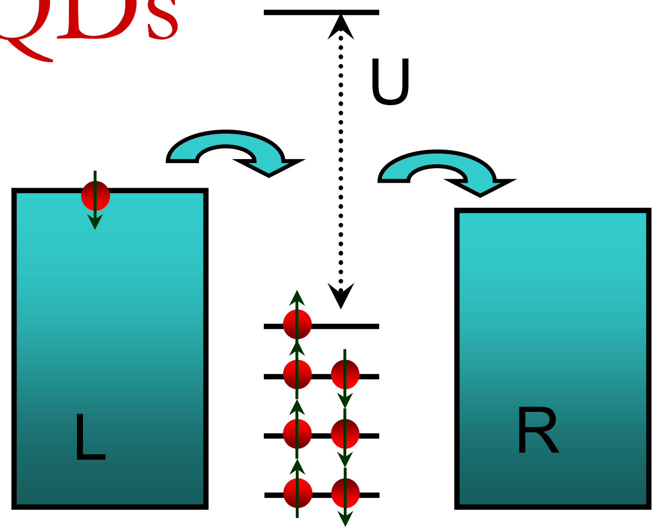
Karyn Le Hur

SU(2) Kondo effects in QDs





$$H_{dot} = \sum_s \varepsilon_d d_s^+ d_s + U n_\uparrow n_\downarrow \qquad H_t = \sum_{i=R/L} t_i \sum_{k,s} (c_{k,is}^+ d_s + h.c.)$$

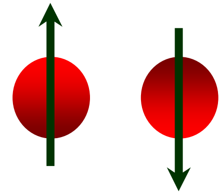
- Charge excitation quenched: quantum dot acts as an impurity ½-spin.
- Tunneling to leads: exchange interaction between conduction electrons and spin impurity.

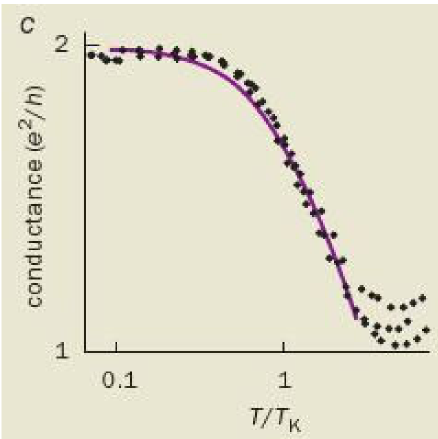
$$H_K = J \vec{S}_i . (c_s^+ \vec{\sigma}_{ss'} c_{s'})$$

SU(2) Kondo effects in QDs

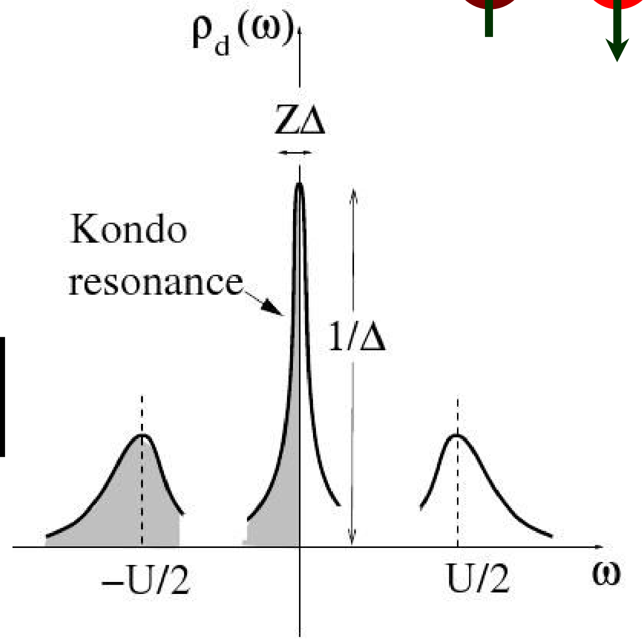
Local interaction $$H_K = J\vec{S}_i.(c_s^{+}\vec{\sigma}_{ss'}c_{s'})$$

- Perturbation breaks down at low energy (J effectively large).
- Formation of many-body singlet for ground state.
- Enhanced scattering and conductance (Kondo resonance).



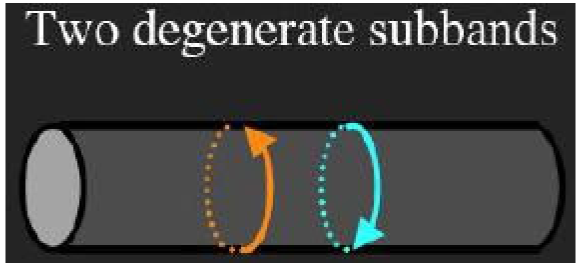


$$T_K = D e^{-1/2\nu J}$$



L. Kouwenhoven and L. Glazman (2001)

Orbital +spin: SU(4) Kondo ?

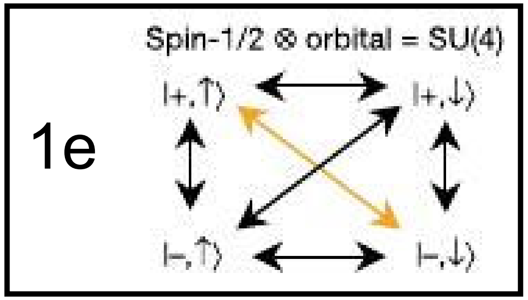


- Subbands: SU(4) symmetry for orbital-selective tunneling.

  Choi, Lopez, Aguado (PRL 2005)

- Electron exchange couples all states
  - Much larger Kondo temperature

$$T_K^{SU(N)} = D\, e^{-1/N\nu J}$$

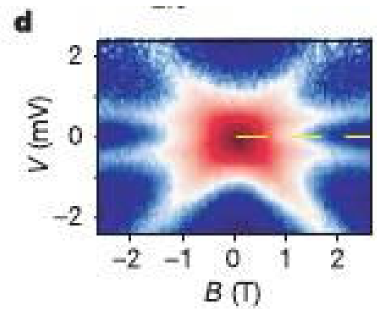


- Strong competition with two-level SU(2),

SU(4) unstable (asymmetry or non-orb-sel-tun)

Lim et al (PRB 2006)

- Four peaks splitting in magnetic field



Jarillo-Herrero (Nature 2005)

Local Fermi liquid picture

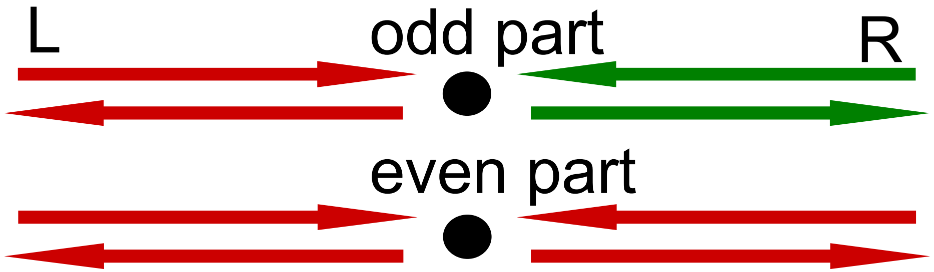
Nozières (1974)

- Quasiparticle phase-shift:

$$\varepsilon << T_K$$

$$\delta_\sigma(\varepsilon) = \underbrace{\frac{\pi}{2} + \frac{\alpha_1 \varepsilon}{T_K}}_{\text{Elastic}} - \underbrace{\frac{\phi_1 n_{-\sigma}}{\nu T_K}}_{\text{Inelastic}}$$

$$S(k) = e^{2i\delta(\varepsilon_k)}$$



$$e^{i(k_F+k)x} - e^{-i(k_F+k)x}$$

$$e^{i(k_F+k)x} - S(k)e^{-i(k_F+k)x}$$

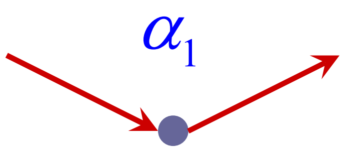
$x < 0$

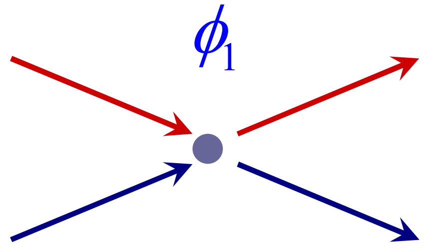
- $\delta = 0$, L/R leads separated: $G = 0$
- $\delta = \pi/2$, plane waves for scattering states $G = 2e^2/h$

Local Fermi liquid picture

Nozières (1974)

$$\delta_\sigma(\varepsilon) = \frac{\pi}{2} + \frac{\alpha_1\,\varepsilon}{T_K} - \frac{\phi_1\,n_{-\sigma}}{\nu T_K}$$





- Scattering via virtual polarization of singlet (energy T_K).
- δ(0) imposed by Friedel sum rule.
- Floating of Kondo resonance: $\alpha_1 = (N-1)\,\varphi_1$
- Justification: conformal theory + Bethe-Ansatz solution, describes vicinity of the strong coupling fixed point.

Affleck, Ludwig (1991)

Lesage, Saleur (PRL, 1999)

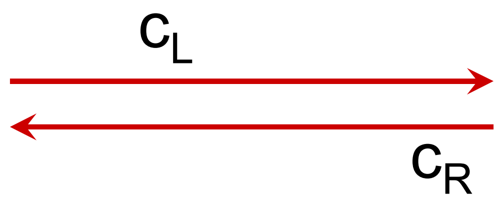
Bazhanov, Lukyanov, Zamolodchikov (Comm. Math. Phys. 1997)

Fractional Shot Noise (T=0, V<<T$_K$)

Sela, Oreg, von Oppen, Koch (PRL, 2007)

- Linear transport: δ(0)=π/2 is sufficient.
  - perfect transmission and no noise.

$$I = I_0 = \frac{2e}{h} eV \qquad S = 0$$

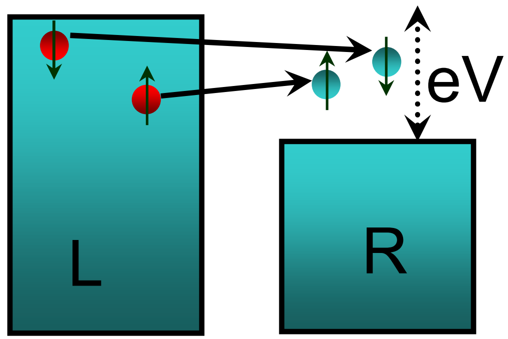


$$b_k = \frac{c_{L,k} + c_{R,k}}{\sqrt{2}}$$

- Non-linear transport (~V³).
  - effective charge e^* =S/ 2 I_{BS} . e^* =e for non-interacting electrons (only elastic scattering).
  - poissonian statistic for backscattering,

events with one/two electrons.

$$H_I = \frac{\phi_1}{\pi \nu^2 T_K} \sum b^\dagger_{k,\uparrow} b_{k',\uparrow} b^\dagger_{q,\downarrow} b_{q',\downarrow} \Rightarrow \frac{e^*}{e} = \frac{5}{3}$$



Gogolin, Komnik (PRL 2006) Golub (PRB 2006) Meir, Golub (PRL 2002)

Current operator

- Straightforward approach: $\hat{I} = G_0 V - \frac{e}{2}\frac{d}{dt}(N_l - N_r)$

  Kaminski, Nazarov, Glazman (PRB 2000)

  - does not apply for T≠0 Noise (Nyquist Noise not recovered)

- Back to Büttiker but with even/odd channels.

$$x < 0 \qquad \psi(x) = \sum_k \frac{e^{i(k_F+k)x} - e^{-i(k_F+k)x}}{\sqrt{2}} a_k + \sum_k \frac{e^{i(k_F+k)x} - S(\varepsilon_k)e^{-i(k_F+k)x}}{\sqrt{2}} b_k$$

- Current operator

$$\hat{I} = \frac{e}{2vh}\left(a^\dagger(x)b(x) - a^\dagger(-x)\hat{S}b(-x) + h.c.\right)$$

$b(x) = \sum_k e^{ikx} b_k$ all elastic scattering encoded in S

Current noise computation

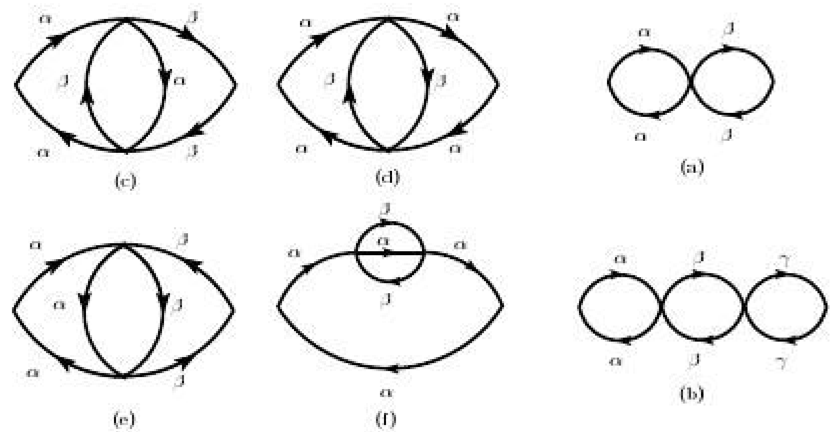
- Landauer-Büttiker recovered with $T(\varepsilon) = \left(1 - \mathrm{Re}\, S(\varepsilon)\right)/2$

$$I = \frac{2e}{h} \int_{\mu_R}^{\mu_L} d\varepsilon\, T(\varepsilon)$$

$$S = \frac{4e^2}{h} \int_{\mu_R}^{\mu_L} d\varepsilon\, T(\varepsilon)\left(1 - T(\varepsilon)\right)$$

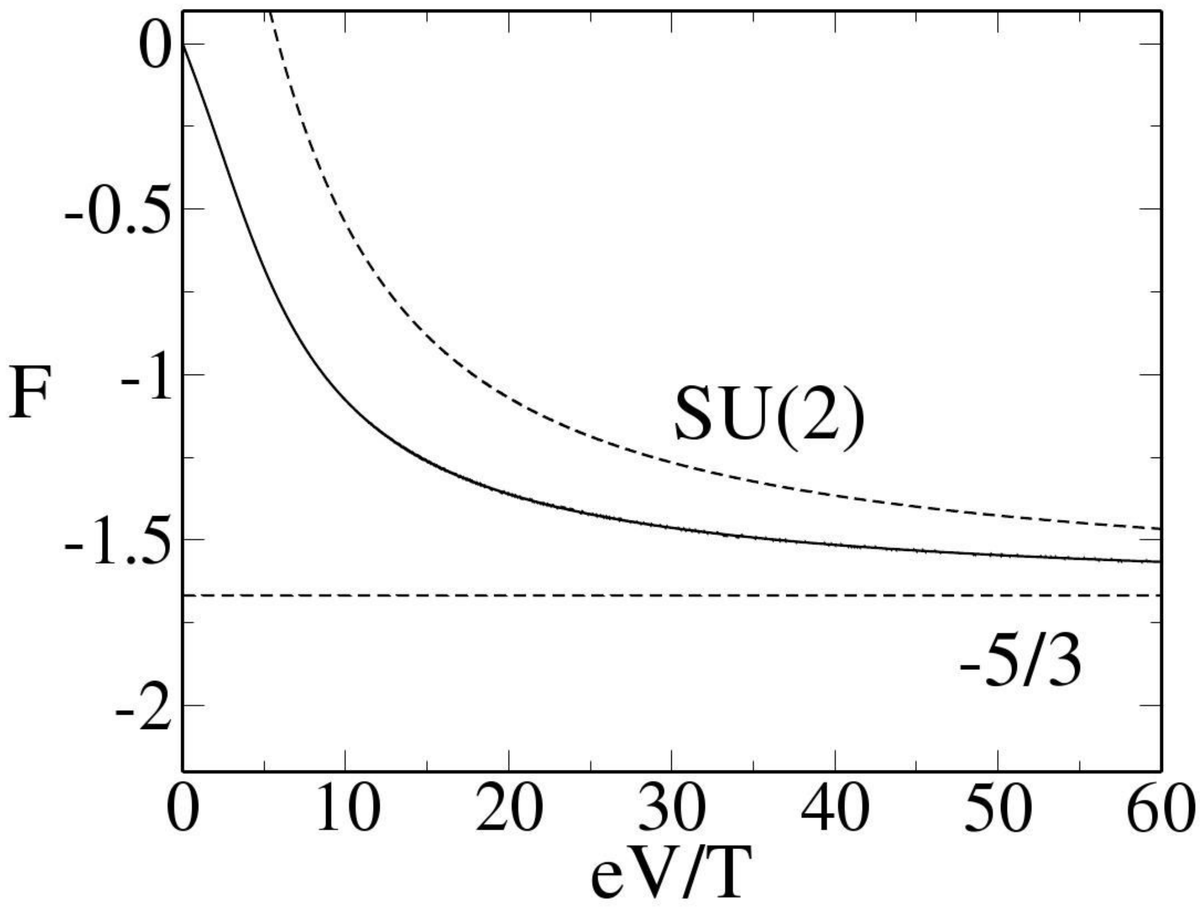
- Interactions on top of that, full perturbative Keldysh calculation is required.

$$H_I = \frac{\phi_1}{\pi \nu^2 T_K} \sum b^{\dagger}_{k,\uparrow} b_{k',\uparrow} b^{\dagger}_{q,\downarrow} b_{q',\downarrow}$$



Fano factor

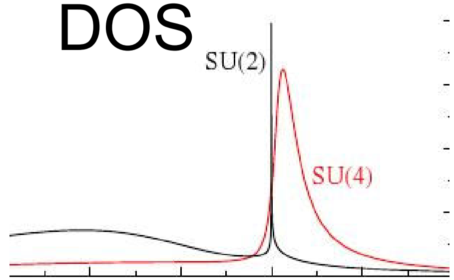
$$F = \frac{1}{2e}\frac{S(V,T) - S_0(V,T) - 4T\partial_V \delta I(V,T)}{I(V,T) - I_0(V)}$$



Mora, Leyronas, Regnault (PRL 2008)

SU(4) at low energy

Lim et al (PRB 2006)



- Friedel sum rule implies $\delta(0)=\pi/4$.
  - Transmission $T_0 = 1/2$
  - Partition noise at strong coupling, $S_0 = 2\, e^3 V/h$, does not vanish like SU(2).
  - Non-linear transport, δI, $\delta S \sim V^3$

- Effective Kondo resonance above Fermi level.
  - model is not p-h symmetric.

- FL description: lowest order.

$$\delta_\sigma(\varepsilon) = \frac{\pi}{4} + \frac{\alpha_1 \varepsilon}{T_K} - \frac{\phi_1 n_{-\sigma}}{\nu T_K}$$

$$\alpha_1 = (N-1)\phi_1$$

Not sufficient for FL corrections !

FL corrections: second generation

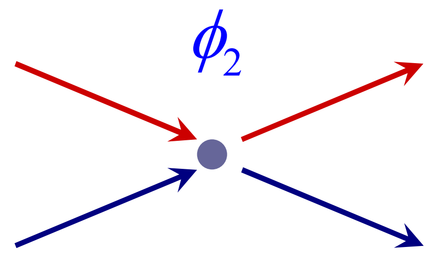
$$\delta_\sigma(\varepsilon) = \frac{\pi}{4} + \frac{\alpha_1\,\varepsilon}{T_K} + \frac{\alpha_2\,\varepsilon^2}{T_K^2}$$

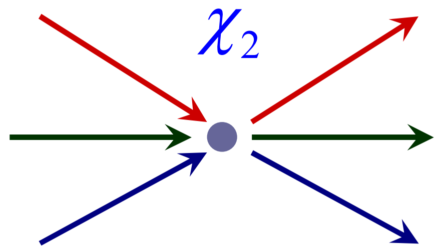
elastic

$$-\frac{\phi_1}{\nu T_K}\sum_{\varepsilon',\sigma'\neq\sigma} n_{\sigma'}(\varepsilon') - \frac{\phi_2}{\nu T_K^2}\sum_{\varepsilon',\sigma'\neq\sigma}\frac{\varepsilon+\varepsilon'}{2} n_{\sigma'}(\varepsilon')$$

$$-\frac{\chi_2}{\nu^2 T_K^2}\sum_{\varepsilon',\varepsilon'',\sigma'<\sigma''} n_{\sigma'}(\varepsilon') n_{\sigma''}(\varepsilon'')$$

inelastic





- Kondo resonance 'floating': $\alpha_2 = \frac{N-1}{4}\phi_2 \quad \phi_2 = (N-2)\,\chi_2$
- Current approach (CFT): $\delta H = -\lambda_2\, d_{ABC} J^A J^B J^C$ 3rd Casimir

Current noise for SU(4)

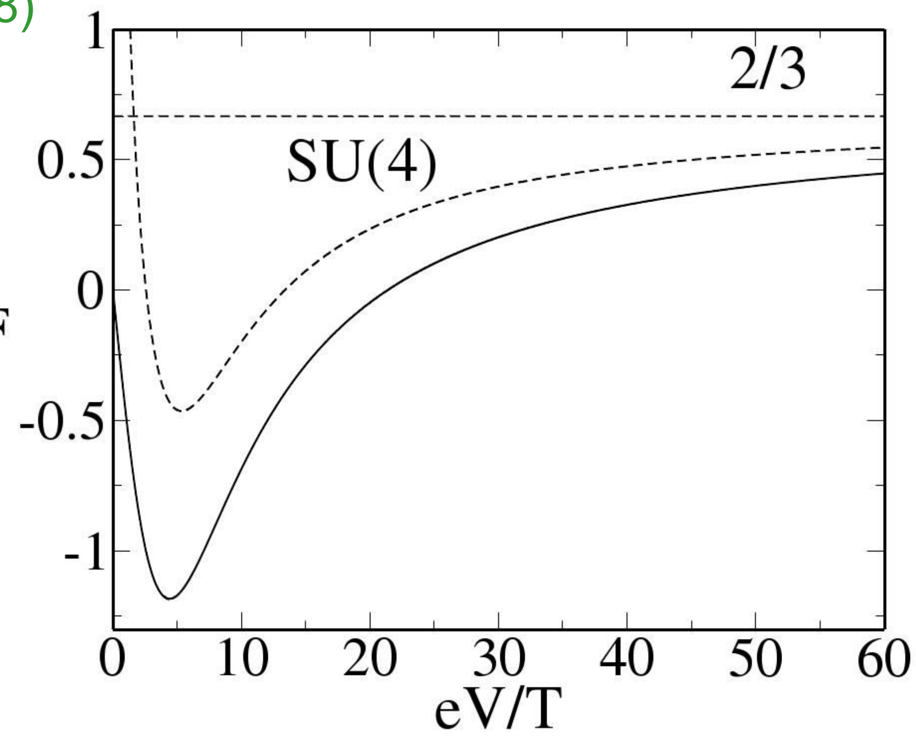
$$F = \frac{1}{2e}\frac{S(V,T) - S_0(V,T) - 4T\partial_V(\delta I)(V,T)}{I(V,T) - I_0(V)}$$

Vitushinsky, Clerk, Le Hur (PRL 2008)

Resonant level at energy ε_K

$$\delta_{el}(\varepsilon) = \arctan\left(\frac{\Gamma}{\varepsilon_K - \varepsilon}\right)$$

$$\Rightarrow \alpha_2 = \alpha_1^2$$



Mora, Leyronas, Regnault (PRL 2008)

Conclusion

- General framework to compute current and noise for SU(N) screening.
- Effective charge characterizes non-linear transport.
- Important temperature corrections to shot noise.

Vitushinsky, Clerk, Le Hur (PRL 2008)

Mora, Leyronas, Regnault (PRL 2008)