



# Rectification and nonlinear transport in chaotic dots and rings



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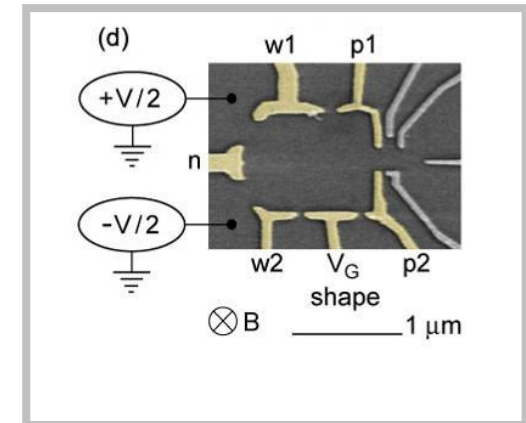
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Thanks for discussions:

Hélène Bouchiat

Renaud Leturcq

Dominik Zumbühl

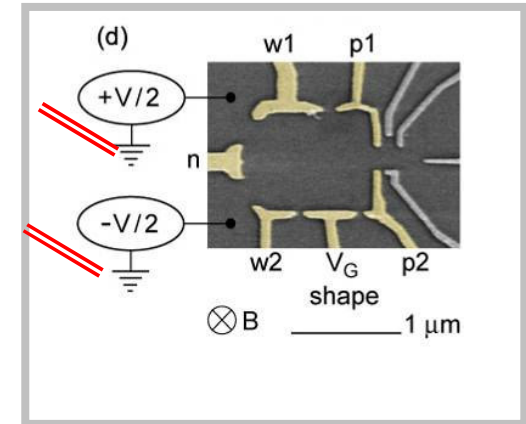


Zumbühl et al'06

La Thuile, 2008

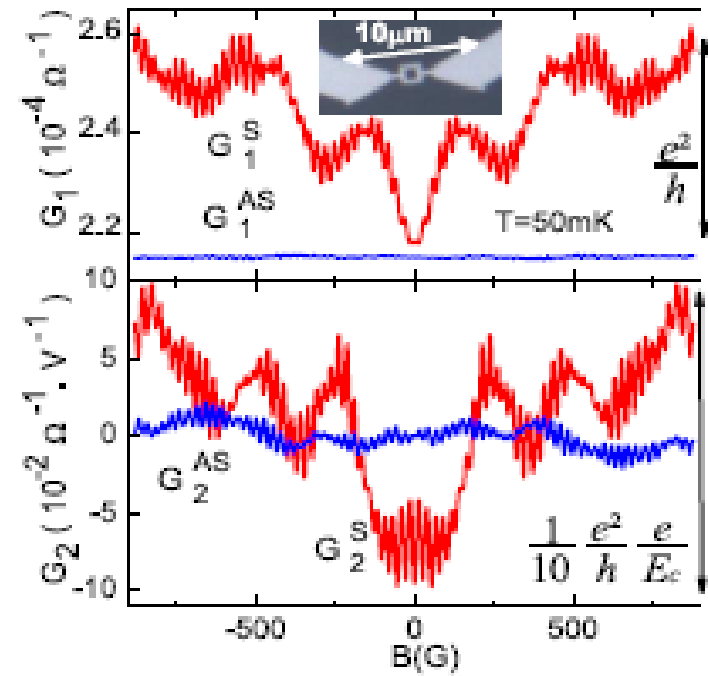
# Outline

- Th./ exp. motivation
- ✓ Non-linear transport: DC  $\rightarrow$  DC
- ✓ Rectification: AC  $\rightarrow$  DC



Zumbühl et al'06

- Importance of bias mode
- Rectification
- Aharonov-Bohm phase



Angers et al'07

# Motivation: Theory

Out-of-equilibrium: "Onsager" is invalid

$$G = I/V \Rightarrow G_{\alpha\beta}(\Phi) \neq G_{\beta\alpha}(-\Phi)$$

$$I \approx GV + \boxed{V^2\mathcal{G}}$$

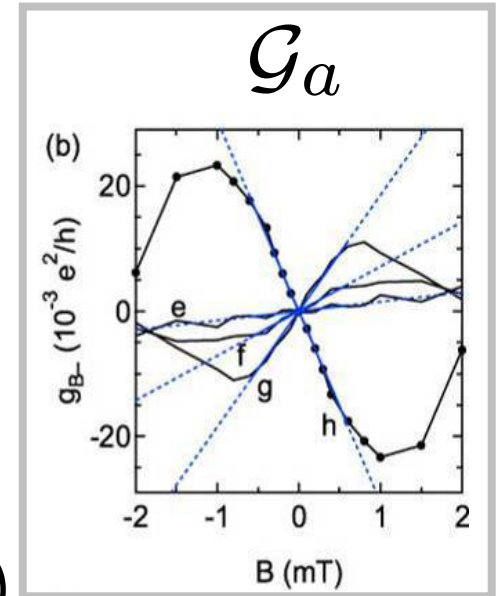
Random and asymmetric,  $I(\Phi) \neq I(-\Phi)$

$$\begin{pmatrix} \mathcal{G}_s \\ \mathcal{G}_a \end{pmatrix} = \frac{\mathcal{G}(\Phi) \pm \mathcal{G}(-\Phi)}{2}$$

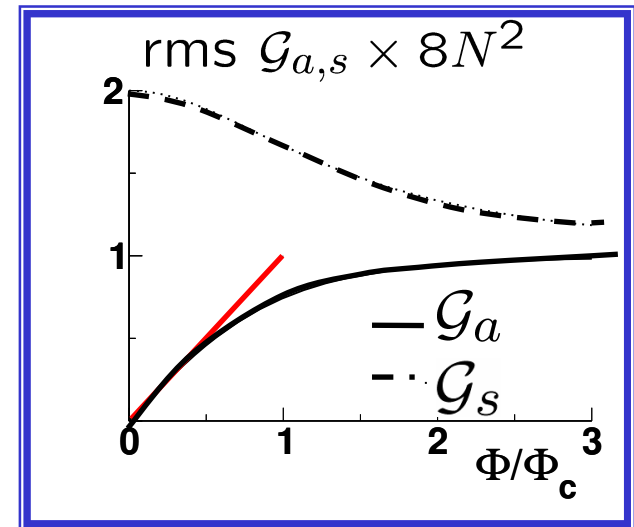
$$\langle \mathcal{G}_a \rangle = \langle \mathcal{G}_s \rangle = 0$$

Non-eq. dot charge is flux-asymmetric

**Electronic interference + interactions**



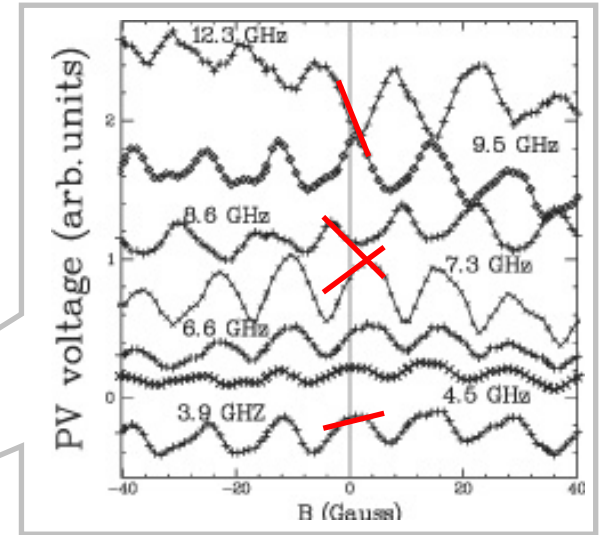
Symmetric dot, strong int.



# Motivation: Experiments

## Non-linear DC $I$ - $V$

- Zumbühl et al '06 (dot)  $\mathcal{G}_a$
- Leturcq et al '06 (AB ring)  $\mathcal{G}_{a,s}$
- Angers et al '07 (AB ring)  $\mathcal{G}_{a,s}$



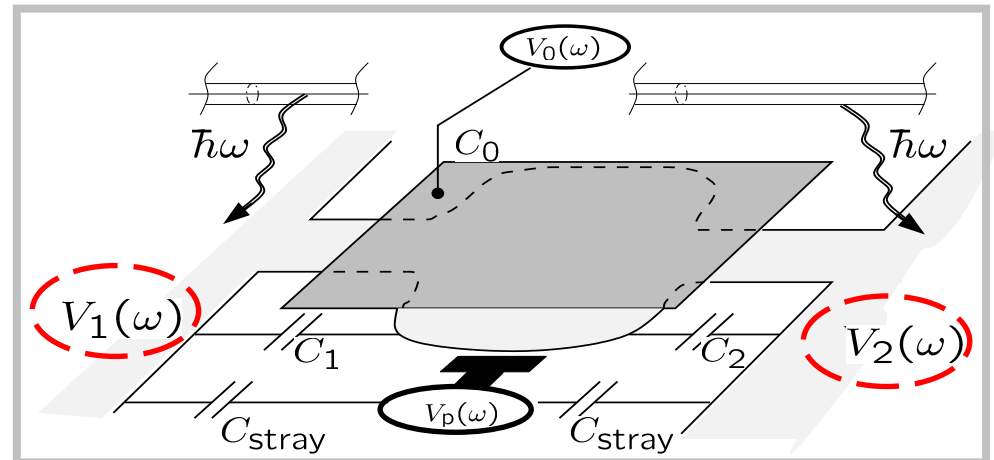
## Photovoltaic effect, AC $\rightarrow$ DC pumping or rectification?

- Switkes et al '99
- DiCarlo et al '05
- Angers et al '07

$$I \propto \overline{V^2 \cos^2 \omega t} \times \mathcal{G}(\omega)$$

Fluctuations of  $\mathcal{G}_{a,s}(\omega)$   
for arbitrary  $\omega$  ?

Phase of AB oscillations?



## Bias mode

Bias mode= how voltages are shifted

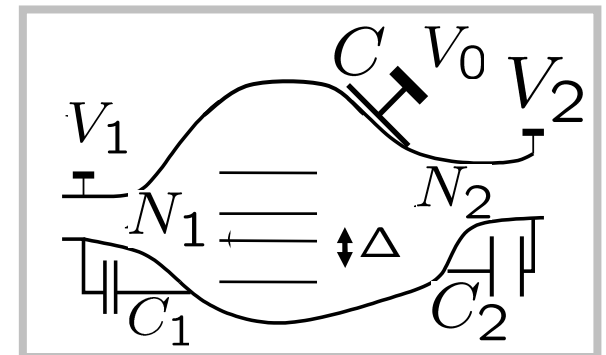
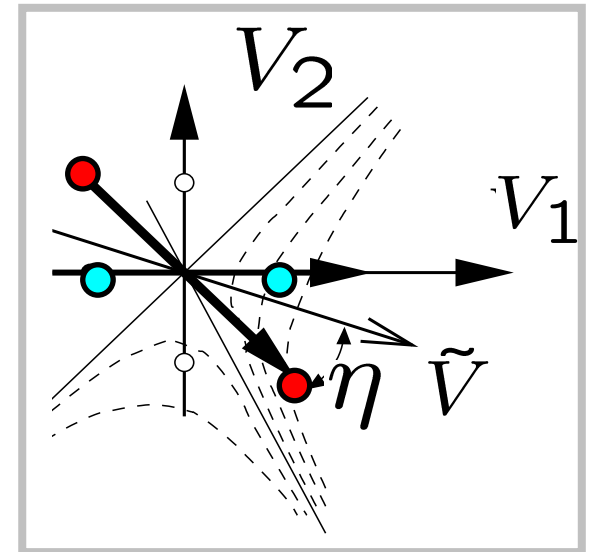
$$I = X V_1^2 + Y V_2^2 + Z V_1 V_2$$

$X, Y, Z$  randomly fluctuate

$\tilde{V}$  is the voltage varied  $\Rightarrow \mathcal{G}(\eta) = \frac{\partial^2 I}{\partial \tilde{V}^2}$

- Shift asymmetrically Zumbühl, Leturcq  
 $\eta = 0 \Rightarrow \tilde{V} = V_1 - V_2$
- Shift one voltage Angers, Marlow, Löfgren  
 $\eta = \pi/4 \Rightarrow \tilde{V} = V_1$

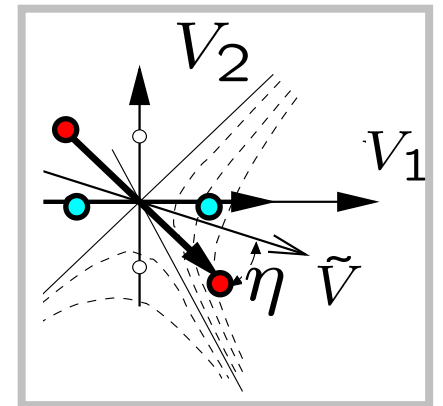
When is the bias mode  $\eta$  important?



# Questions and Approach

- Choose bias mode to maximize  $\mathcal{G}_a/\mathcal{G}_s$
- Consider rectification  $\mathcal{G}_a(\omega), \mathcal{G}_s(\omega)$
- Find phase of AB-oscillations in rings,  $\langle \cotan^2 \phi_{AB} \rangle$

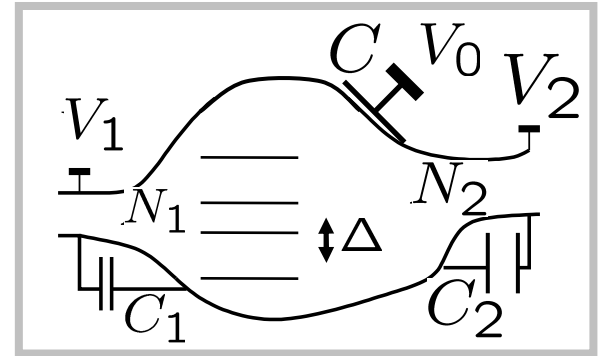
- RMT in open system,  $N \gg 1$ , no CB
- Self-consistent potential,  $U(\{V_i\})$   
(leads to displacement currents)
- Energy-dependent scattering



$$\mathcal{G} = \frac{\partial^2 I}{\partial \tilde{V}^2} \propto \int d\epsilon f'(\epsilon) g'(\epsilon) \left[ \frac{\partial U}{\partial V_0} \tan \eta + \frac{\partial U}{\partial V_2} - \frac{\partial U}{\partial V_1} \right]$$

**random<sub>sym</sub> × [classical<sub>sym</sub> + random<sub>sym+asym</sub>]**  
**(B → -B)**

## Bias mode



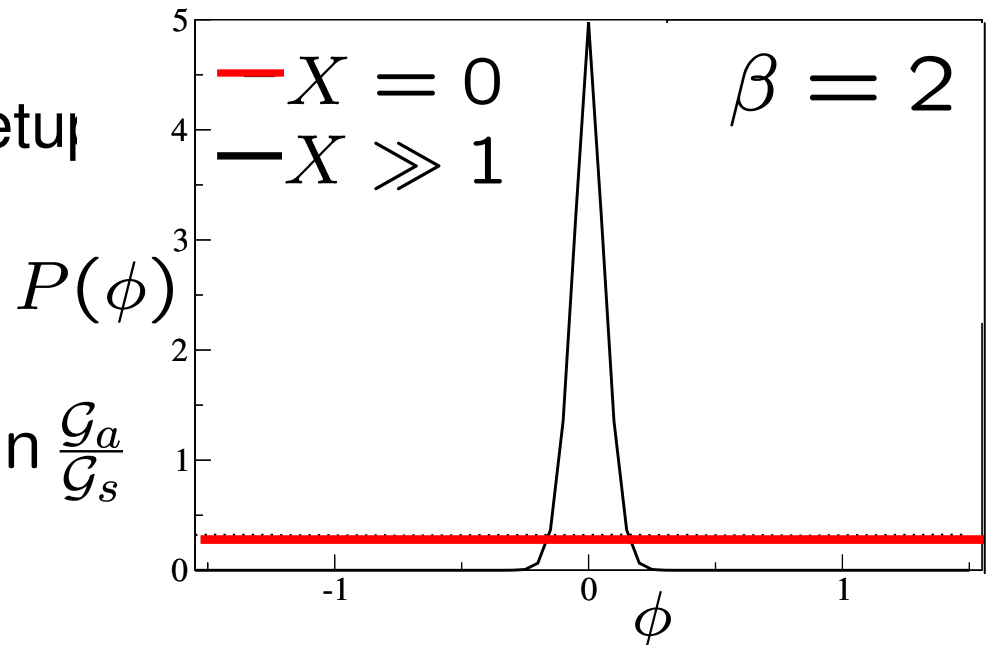
$$\begin{Bmatrix} \mathcal{G}_a^2 \\ \mathcal{G}_s^2 \end{Bmatrix} \propto \left( \frac{C_\mu}{C_\Sigma} \right)^2 \frac{N_1^3 N_2^3}{N^{10}} \left\{ 1 + X \right\}, \quad (T = 0)$$

$$X(\eta) = \frac{N^4}{2N_1 N_2} \left( \frac{C_0 \tan \eta + C_2 - C_1}{2e^2 / \Delta} + \frac{N_2 - N_1}{N} \right)^2$$

To diminish  $\mathcal{G}_s^2$  minimize  $X$  (fluctuations of  $g'(\epsilon) \times \frac{\partial \langle U \rangle}{\partial V_0}$ )

- $\eta \rightarrow 0 \Rightarrow X \rightarrow 0$   
*only for symmetric setup*
- $\eta \neq 0$  compensates setup asymmetry

- Distribution of  $\phi = \arctan \frac{\mathcal{G}_a}{\mathcal{G}_s}$  depends on bias mode  
*...like AB-phase?*

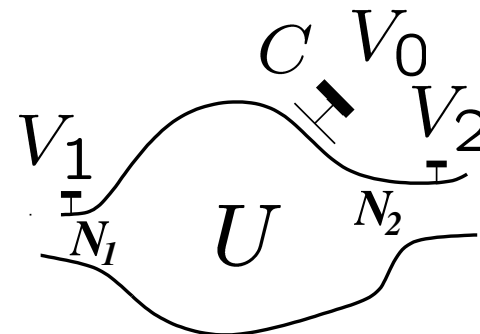


# Rectification (strong interactions)

$$\partial U / \partial V_0 \rightarrow 0, \omega \rightarrow 0$$

$$\partial U / \partial V_0 \rightarrow 1, \omega T_{RC} \gg 1$$

Displacement current is finite, but at large  $\omega$  capacitor is short-cut  $\Rightarrow$  potential  $U$  follows  $V_0$ , not  $V_{1,2}$



$$Z = \frac{1}{i\omega C_\mu} \rightarrow 0, \omega \rightarrow \infty$$

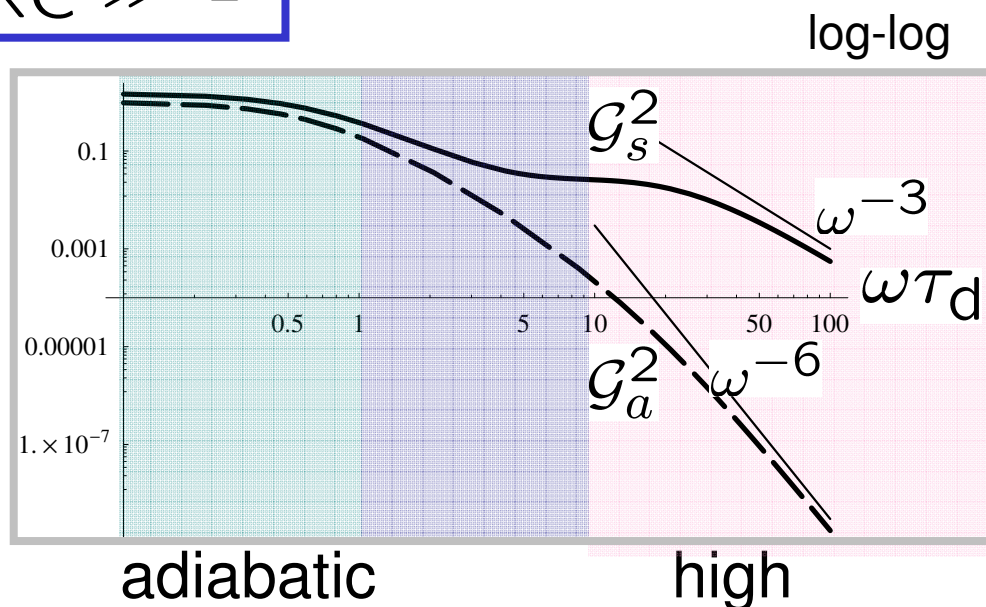
- $\eta \neq 0 \quad \mathcal{G}_s \gg \mathcal{G}_a, \omega T_{RC} \gg 1$

Asymmetry vanishes

Bias mode is more important in AC/DC

- $\eta = 0 \quad \mathcal{G}_a \approx \mathcal{G}_s$

Asymmetry is present





# Toy model of AB-ring

$$\mathcal{G} \approx \mathcal{G}_0 + \mathcal{G}_{AB} \cos\left(2\pi \frac{\Phi}{\Phi_0} + \phi_{AB}\right) \quad \cotan \phi_{AB} = \frac{\mathcal{G}_{s,AB}}{\mathcal{G}_{a,AB}}$$

## Model: chaos+ AB-effect

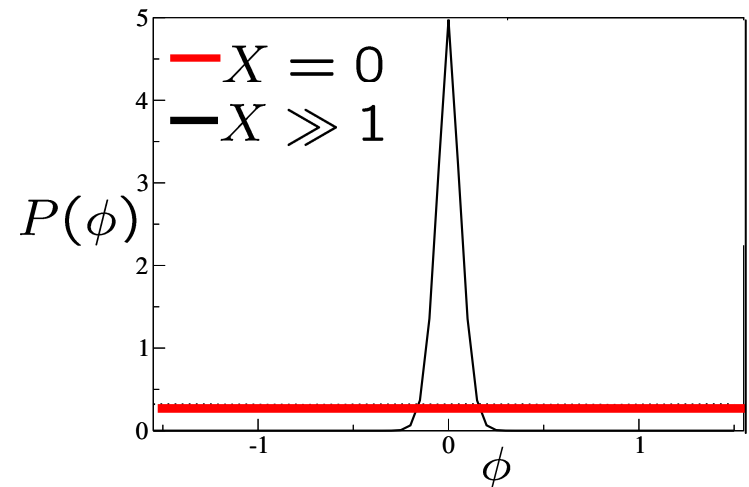
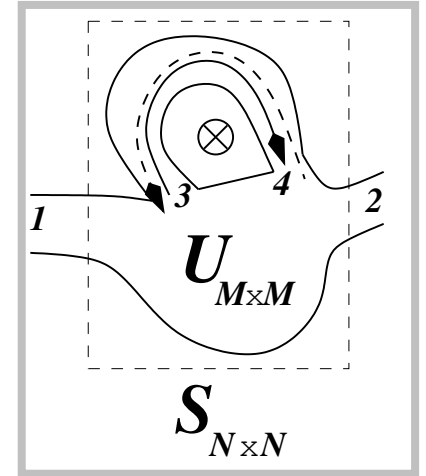
Chaotic dot + narrow ballistic arm

$$\Phi_{\text{arm}} \ll \Phi_{\text{dot}} \ll \Phi_{\text{hole}}$$

$$\langle \cotan^2 \phi \rangle = 1 + 8X(\eta) \begin{cases} 1/5, & T \ll N\Delta/2\pi, \\ 3T/N\Delta, & T \gg N\Delta/2\pi \end{cases}$$

$$X(\eta) = \frac{1}{2} \left( \frac{N\Delta}{e^2/C} \tan \eta \right)^2$$

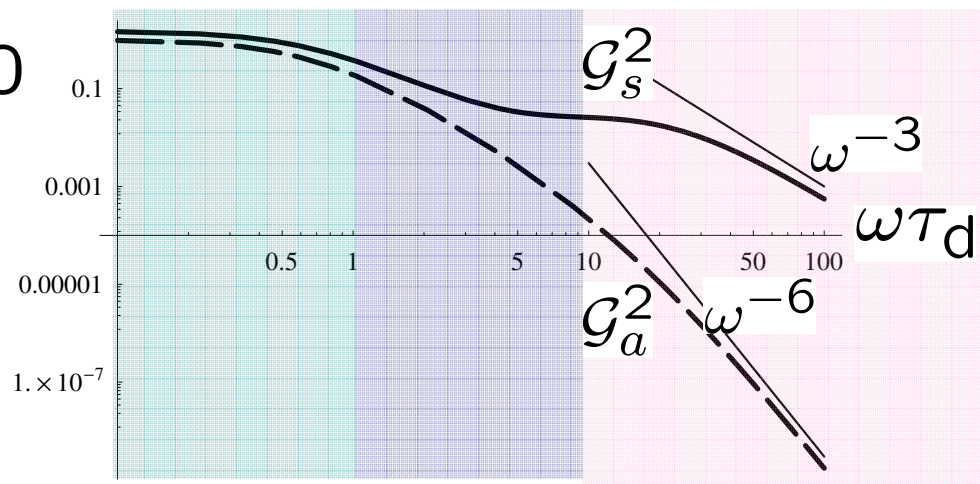
**Qualitatively distribution  
of AB-phase is sensitive  
to bias mode**



# Conclusions

- Bias mode strongly affects measurement of  $\mathcal{G}_s$

- 1) AC-bias with  $\omega \neq 0$
- 2) asymmetric setup
- 3)  $\frac{e^2/C}{\Delta} \neq \infty$



- Found magnetic (a) symmetry in rectification
- In rings found AB-phase,  $\langle \cotan^2 \phi \rangle \sim 1$

??? Frequency mixing, Coulomb in FCS ( $\tau \neq \infty$ )

M.L. Polianski and M. Büttiker, *Phys. Rev. B* **76**, 205308 (2007)  
*Physica E* **40**, 67 (2007)