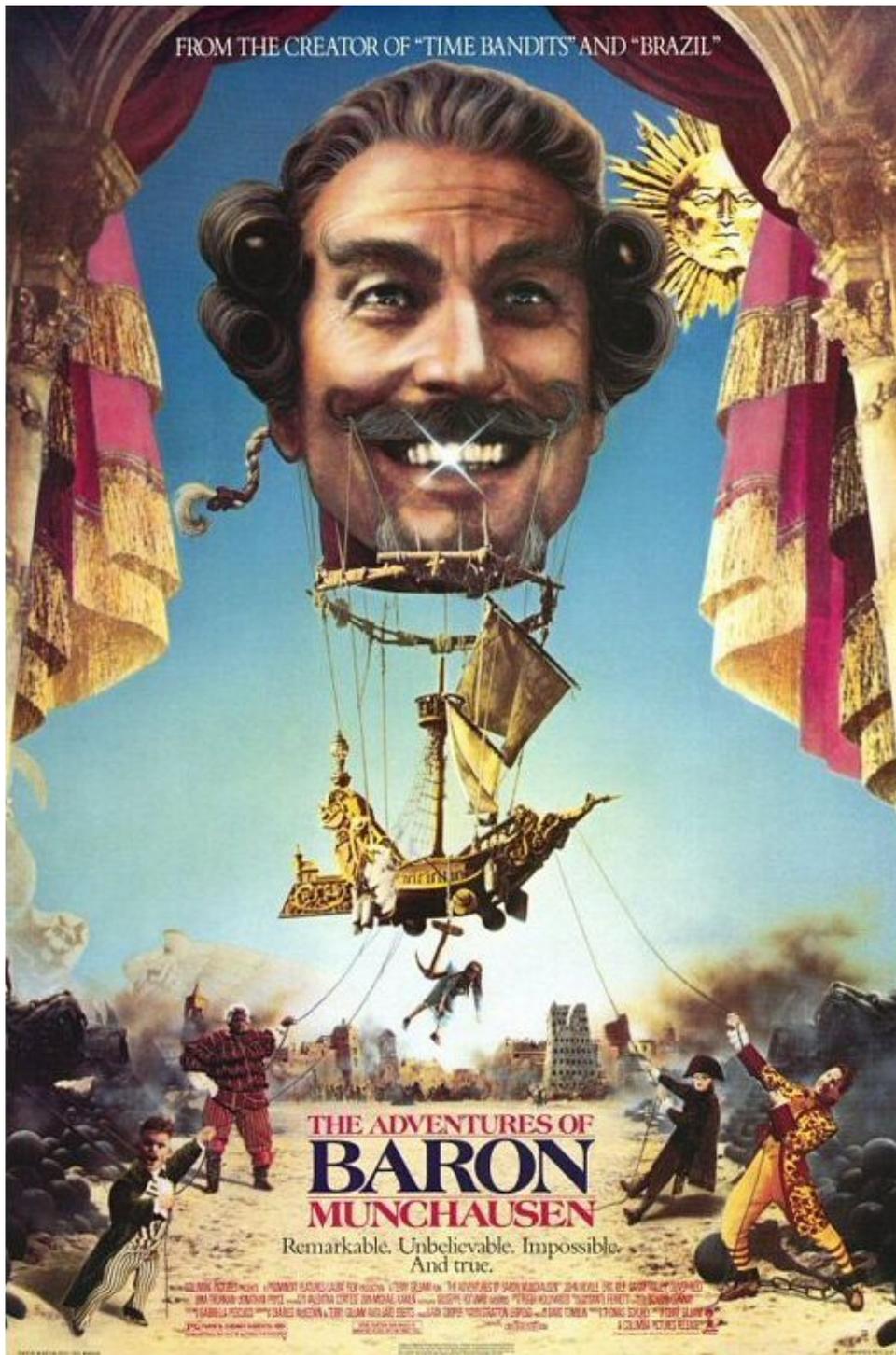


Münchhausen effect,
tunneling in an asymmetric
SQUID



Quantum instability in a dynamically asymmetric dc-SQUID

Theoretische Physik, ETH Zürich, Switzerland

with

Gianni Blatter
Alexander Thomann



Why can't you understand that
Baron Münchhausen is famous
not because he flew
or did not fly to the moon,
but because he doesn't lie

General idea: Münchhausen

Can a heavy particle tunnel?

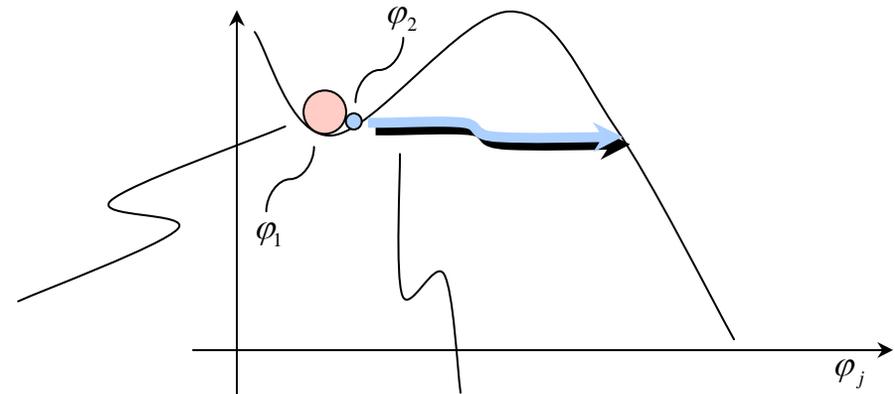
Zero temperature: No thermal excitation.

Yes - if it has a light part which drags the heavy part!

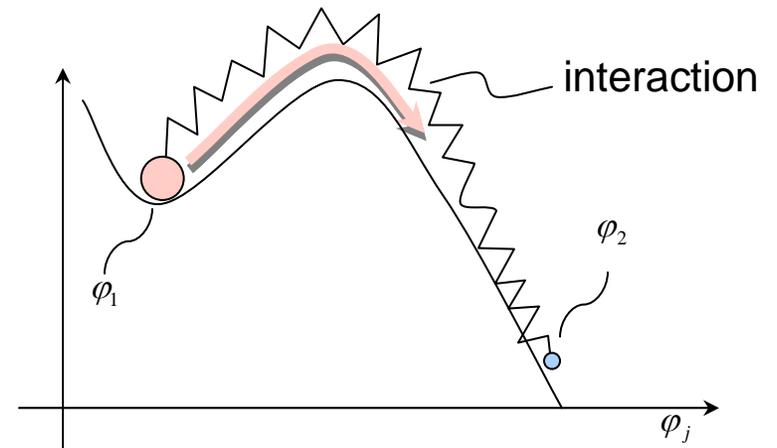
Composite system is in a metastable state

System is unstable w.r.t. **macroscopic quantum tunneling** of the light degree of freedom φ_2

After tunneling the trapping potential for heavy object is distorted and the state may become unstable.



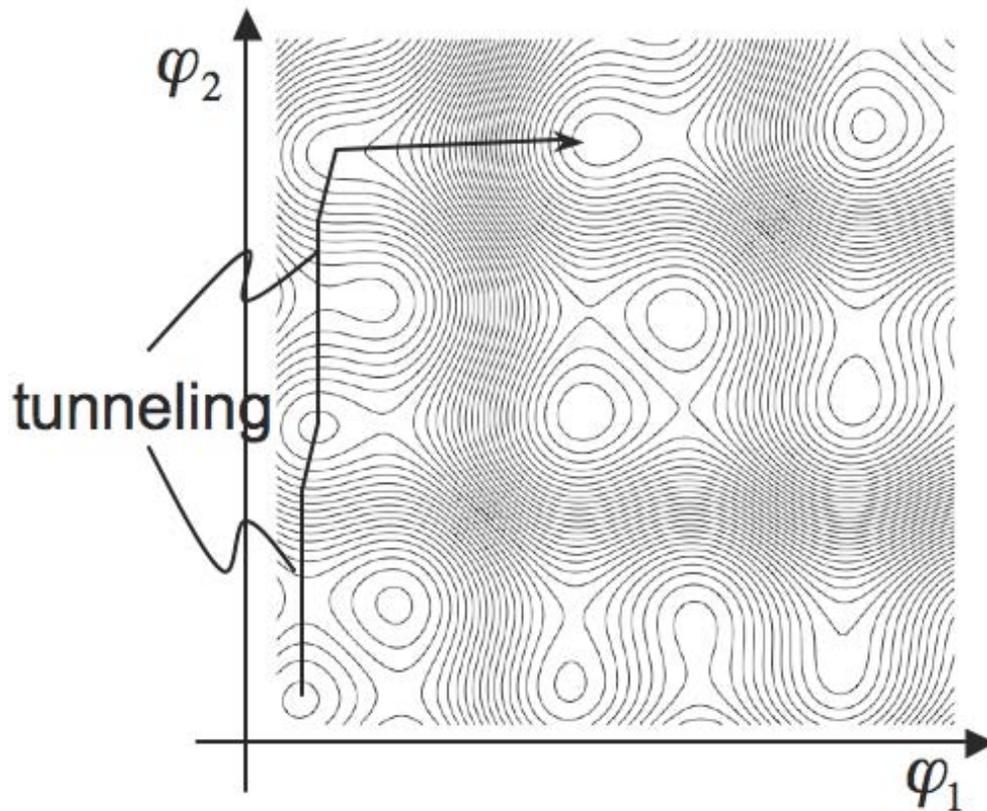
Light part tunnels out of metastable minimum.



And drags the heavy part!

2d potential

If φ_1 and φ_2 are coordinate of the heavy and light parts we have effectively tunneling of a single particle in a 2d potential with strongly anisotropic mass.



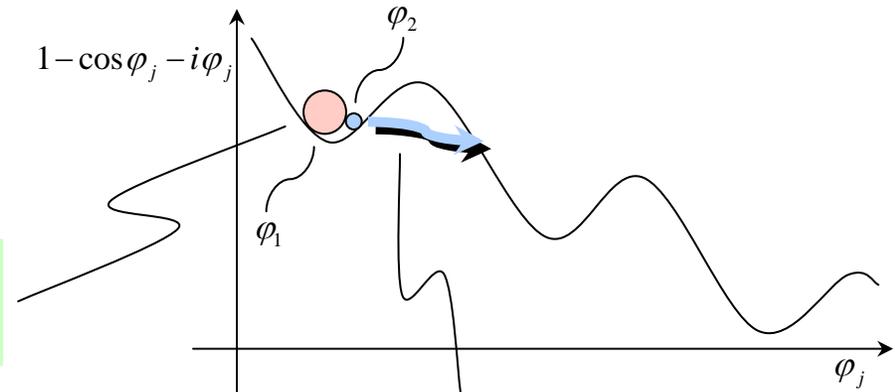
How to realize it ?

SQUID prepared in a **symmetric state** with no external magnetic flux: $\varphi_1 = \varphi_2$

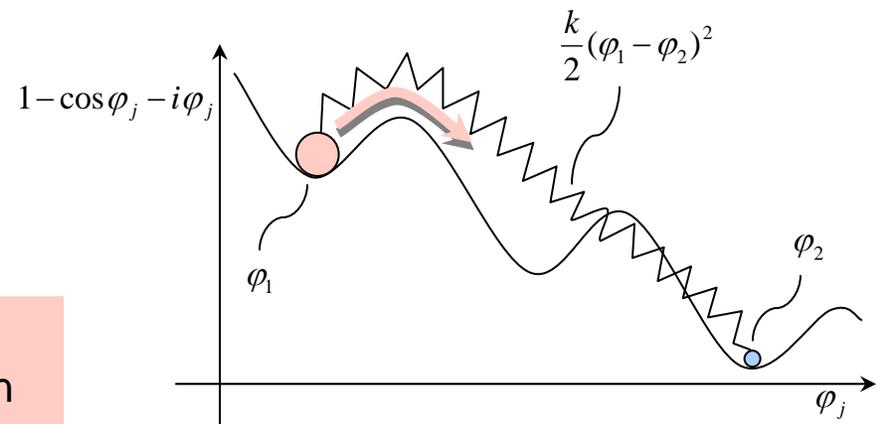
Zero temperature: No thermal excitation.

System is unstable w.r.t. **macroscopic quantum tunneling** of the phase of the small junction φ_2 .

For what bias I does the system exhibit a state of finite voltage over the heavy junction at given coupling (inductance L^{-1})?



Small junction tunnels out of metastable minimum.



Is large junction delocalized?

Setup: Capacitance-asymmetry

dc-SQUID: Two Josephson-junctions in a biased (current I) superconducting ring with total inductance L .

Junctions described by **RCSJ-model** with inductive coupling, (φ_j =phase difference across junction j)

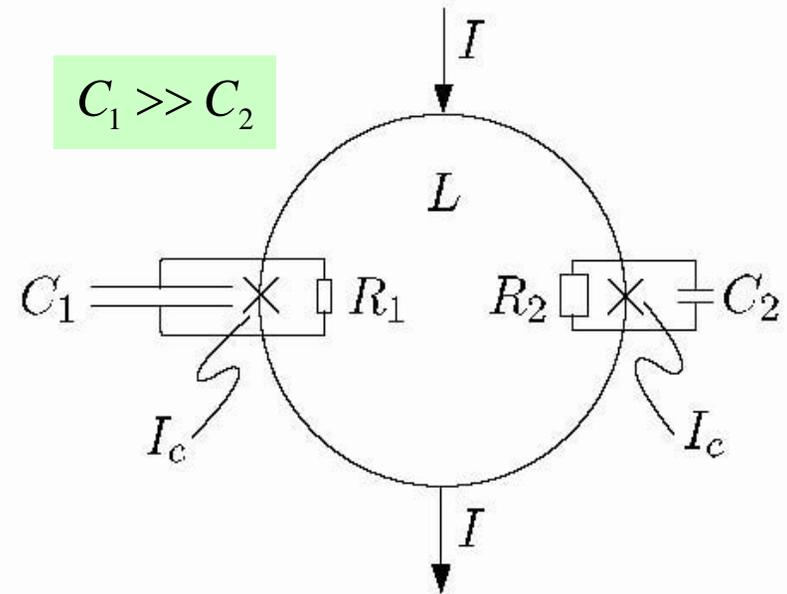
$$m_j \ddot{\varphi}_j + \eta_j \dot{\varphi}_j = -\partial_{\varphi_j} v(\varphi_1, \varphi_2),$$

$$m_j = \frac{\Phi_0}{2\pi I_c} C_j, \quad \eta_j = \frac{\Phi_0}{2\pi I_c} \frac{1}{R_j}.$$

and the interferometer potential

$$v(\varphi_1, \varphi_2) = 1 - \cos \varphi_1 + 1 - \cos \varphi_2 - i(\varphi_1 + \varphi_2) + \frac{k}{2}(\varphi_1 - \varphi_2)^2.$$

$$i = \frac{I}{2I_c}, \quad k = \frac{\hbar}{2\pi I_c} \frac{1}{L},$$



Two junctions in different regimes:

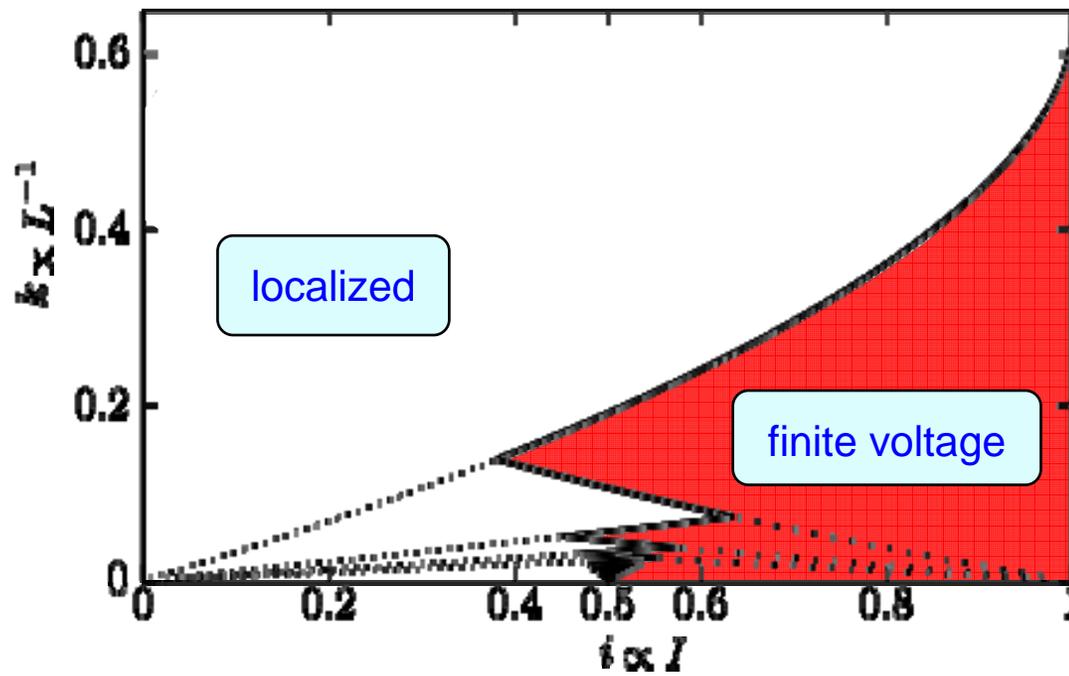
$$\frac{\hbar}{2e} I_c = E_J \gg E_{c1} = \frac{e^2}{2C_1}$$

$$E_J \gtrsim E_{c2}$$

classical regime

quasi-classical regime

Phase diagram overdamped



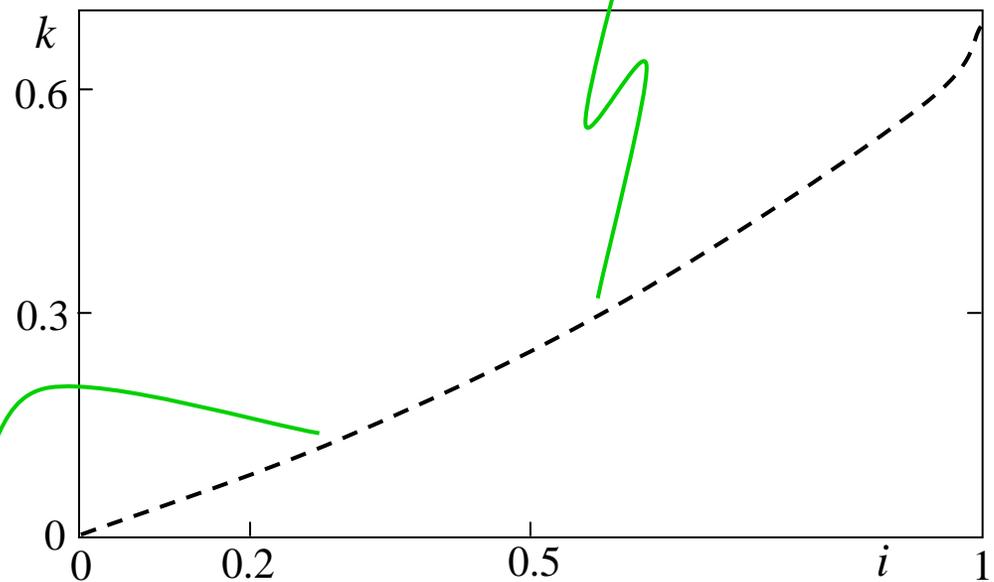
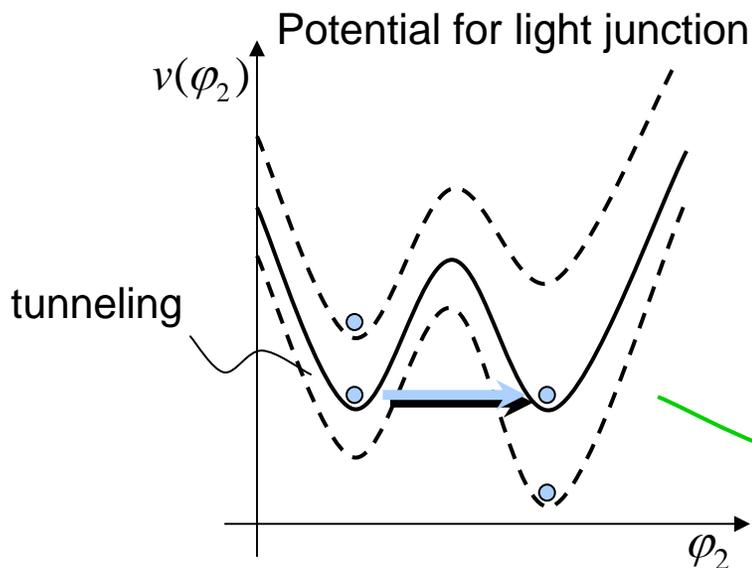
Phase diagram: Strong dissipation

$$\alpha_2 = \eta_2 2\pi E_J / \hbar > 1.$$

Strong dissipation for small junction.

- Total system is in given local minimum of 2d-SQUID potential.
- With increase of current i the next minimum for quantum junction is lowered below the original minimum.

$$k < i / (\pi - \arcsin i)$$



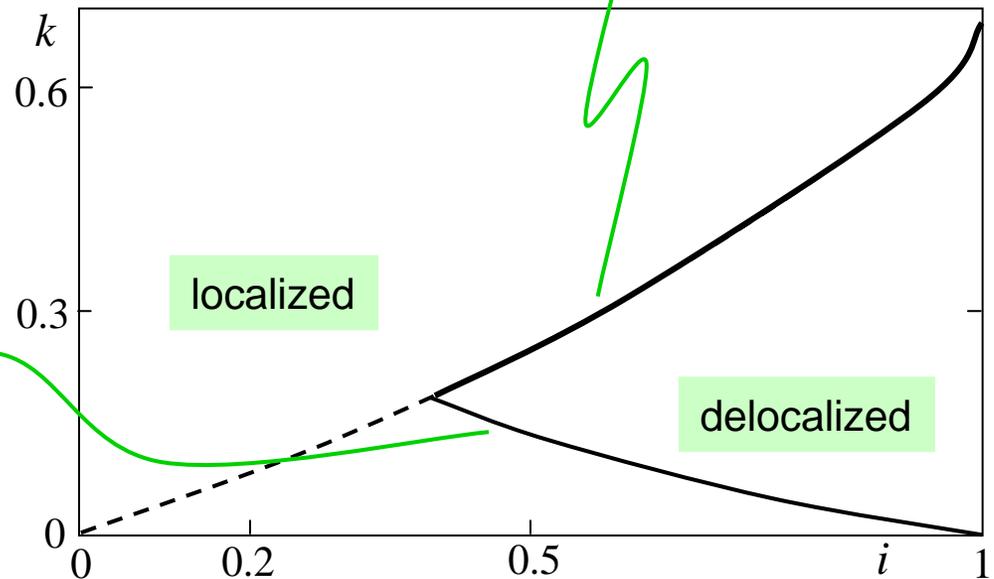
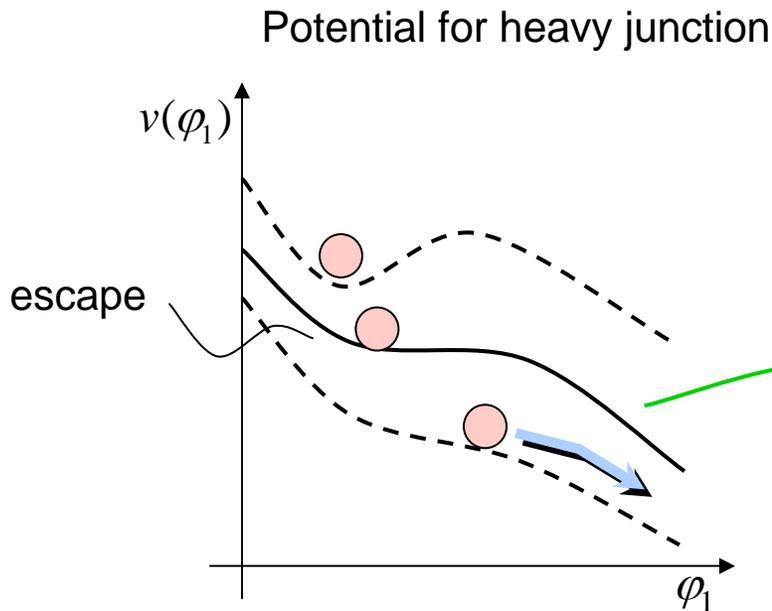
Phase diagram: Strong dissipation

$$\alpha_2 = \eta_2 2\pi E_J / \hbar > 1.$$

Strong dissipation for small junction.

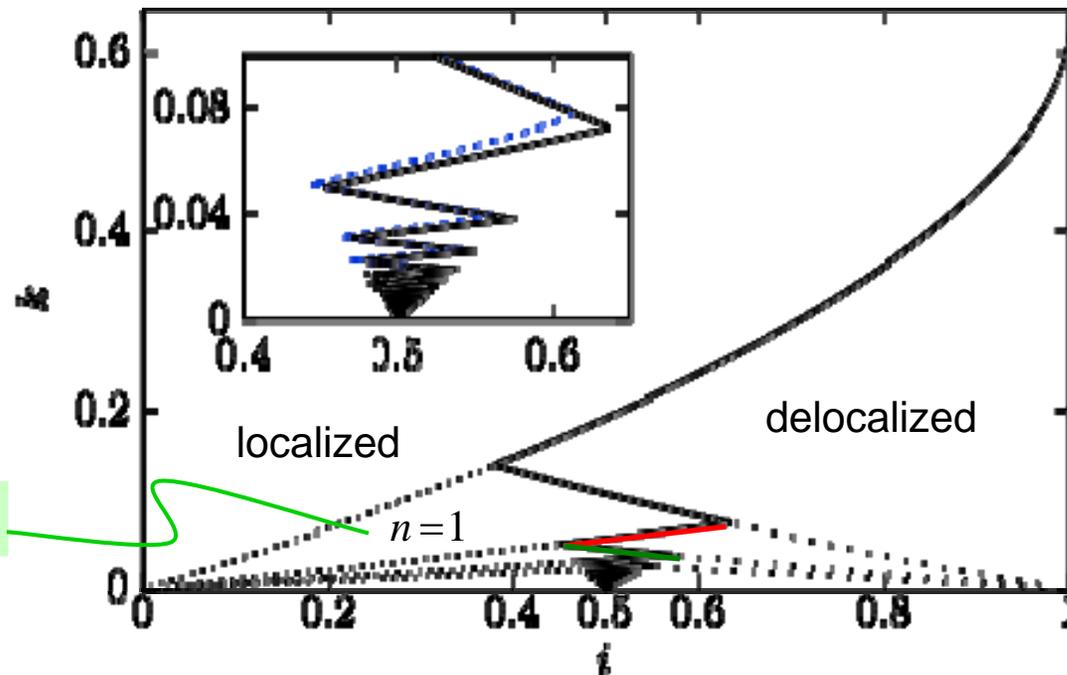
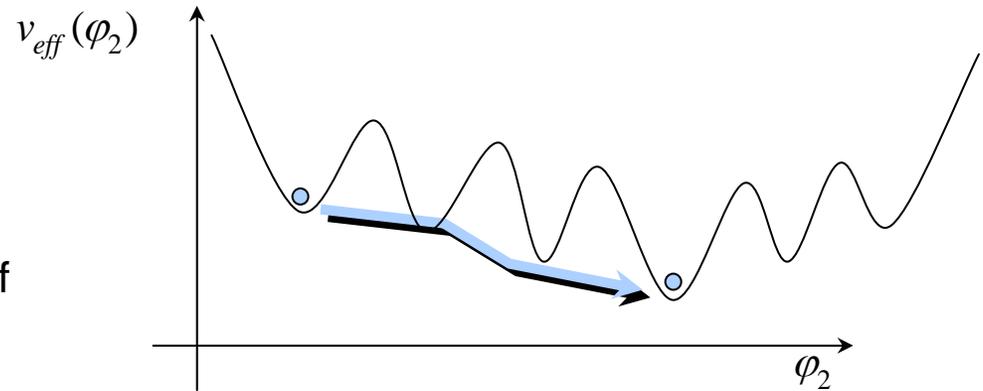
- Total system is in given local minimum of 2d-SQUID potential.
- Large junction is delocalized if minimum becomes unstable.

$$k < i / (\pi - \arcsin i)$$



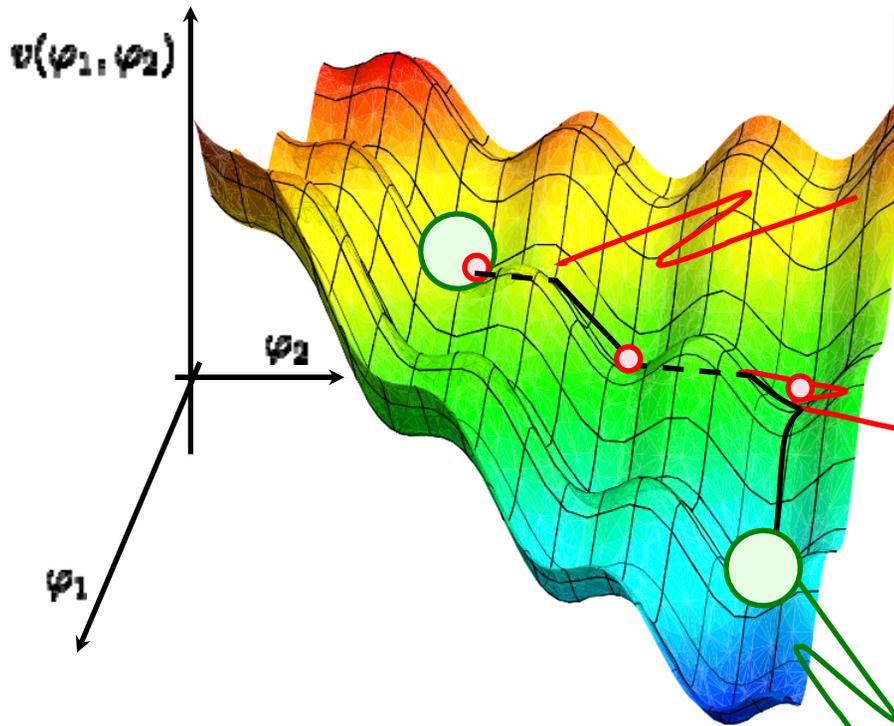
Phase diagram: Strong dissipation

- Every time weak junction tunnels flux enters inside the SQUID loop.
- Small junction will come to rest at global minimum of effective potential.
- Total system is in given local minimum of 2d-SQUID potential.
- Large junction is delocalized if minimum becomes unstable.



no. of flux quanta in the ring.

Decay process initial stage



Quantum tunneling of QJ, phase slip: Approximate flux unit enters.

More flux enters until ground state is reached.

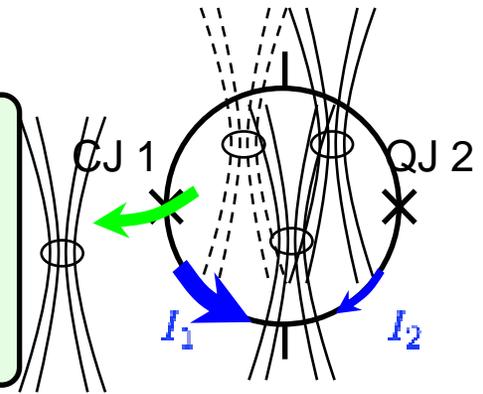
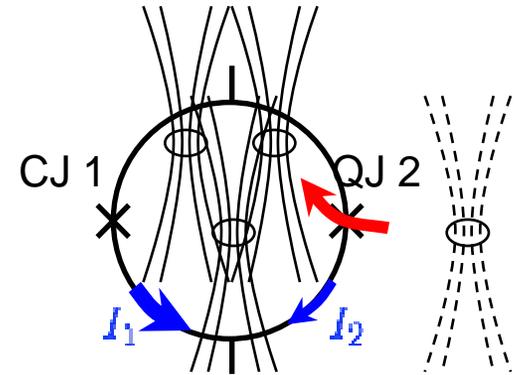
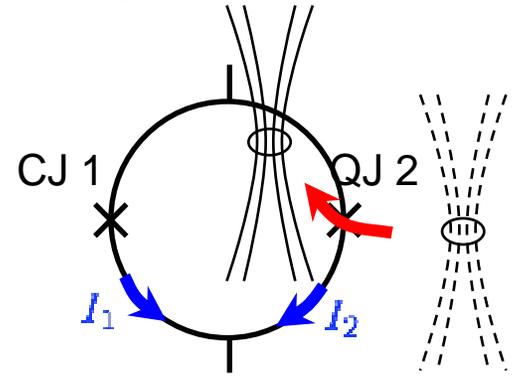
overdamped junctions:

$$\alpha_1, \alpha_2 > 1$$

$$\alpha = (2RC\omega_p)^{-1}$$

$$\hbar^2\omega_p^2 = 8E_J E_C$$

CJ overcritical, flux leaves through CJ upon classical relaxation.

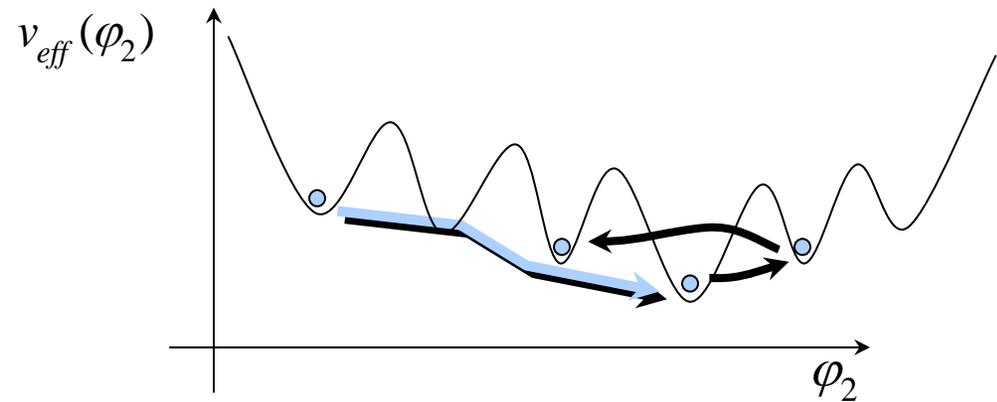


Weak dissipation

$$\alpha_2 = \eta_2 2\pi E_J / \hbar < 1.$$

weak dissipation for small junction.

- Weak junction is delocalized between several wells



Massive dynamics fast ramping

- We consider **instantaneous** ramping

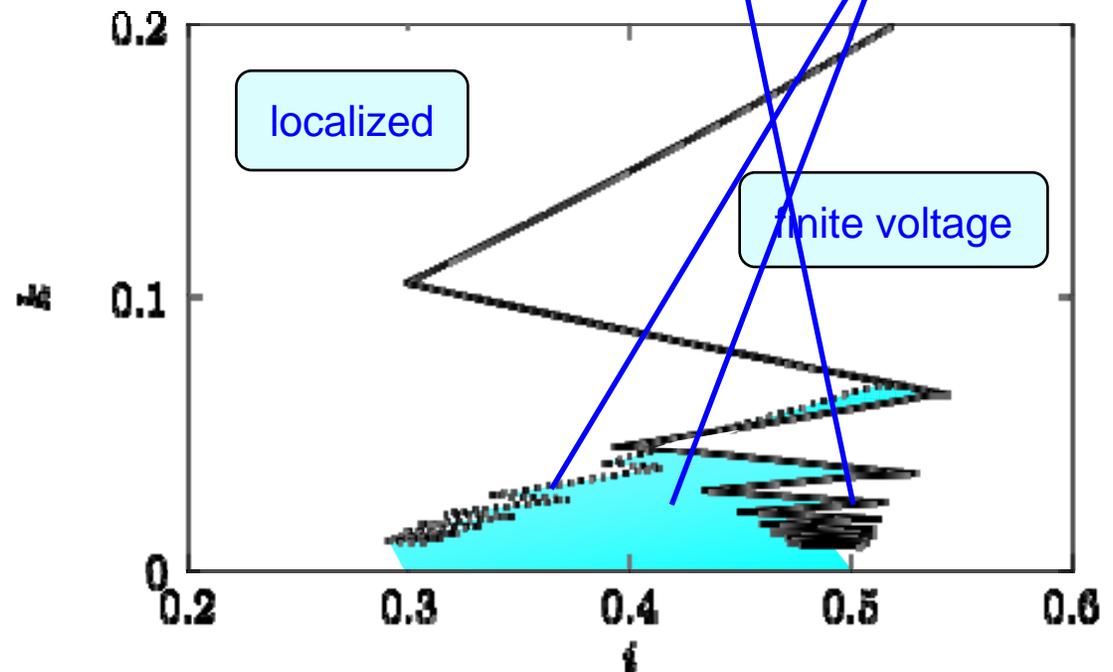
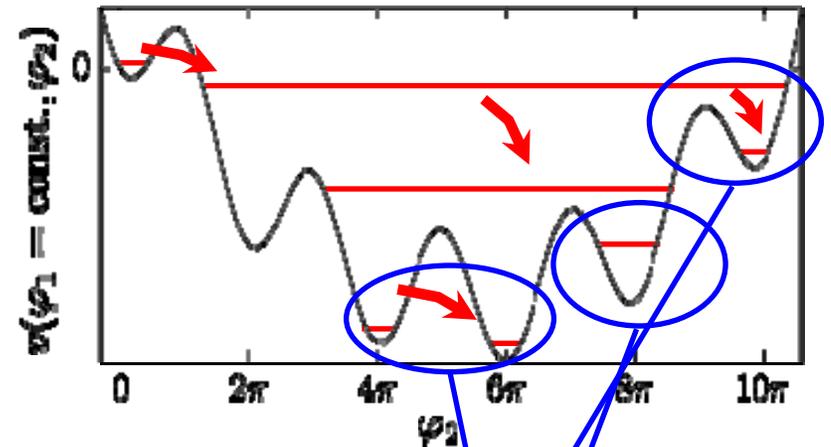
- Relevant residual dissipation:
Assume relaxation in initial/side wells before decay of QJ (can be made consistent with experimental parameters).

- relevant time scales:

$$\Delta_2 \lesssim \omega_{p1} \ll \omega_{p2}$$

- QJ may be trapped in **any** local minimum for a significant amount of time.

- smearing of decay line.



Parameters

RCSJ-model: Parameters:

“mass” $m_J = \frac{\Phi_0 C_J}{2\pi c I_c}$

damping parameter $\eta_J = \frac{\Phi_0}{2\pi c I_c R_J}$

dimensionless bias current $i = \frac{I}{2I_c}$

coupling constant $k = \frac{\Phi_0 c}{2\pi I_c L}$

Realistic parameters:

$C_1 \approx 1\text{pF}$ $C_2 \approx 3\text{fF}$ $I_c \approx 550\text{nA}$ $L \approx 4 - 8\text{nH}$

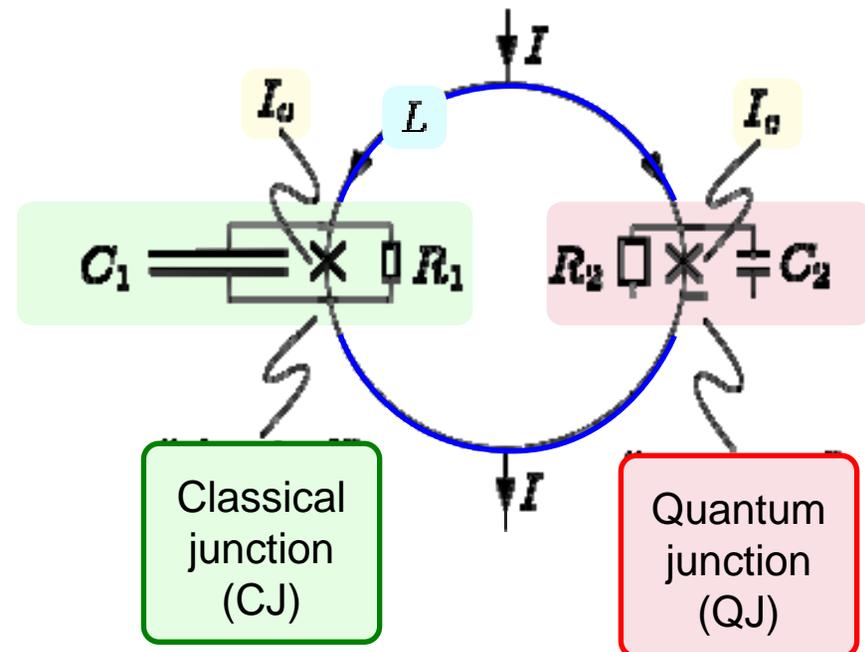
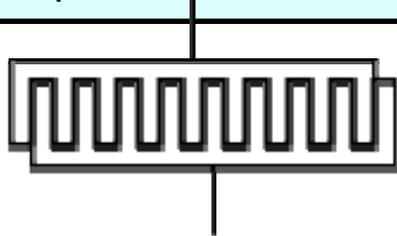
$E_J/E_{c2} \approx 40$ (~ 4-5 levels)

$m_1/m_2 \approx 330$

$k \approx 0.07 - 0.15$

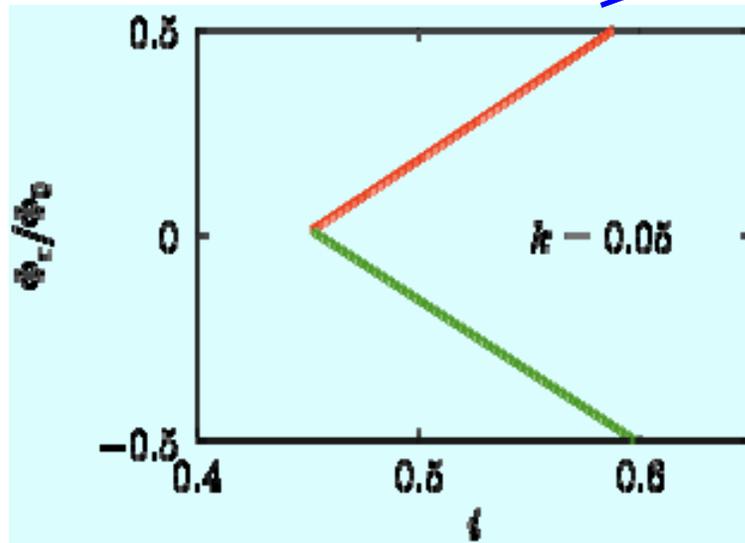
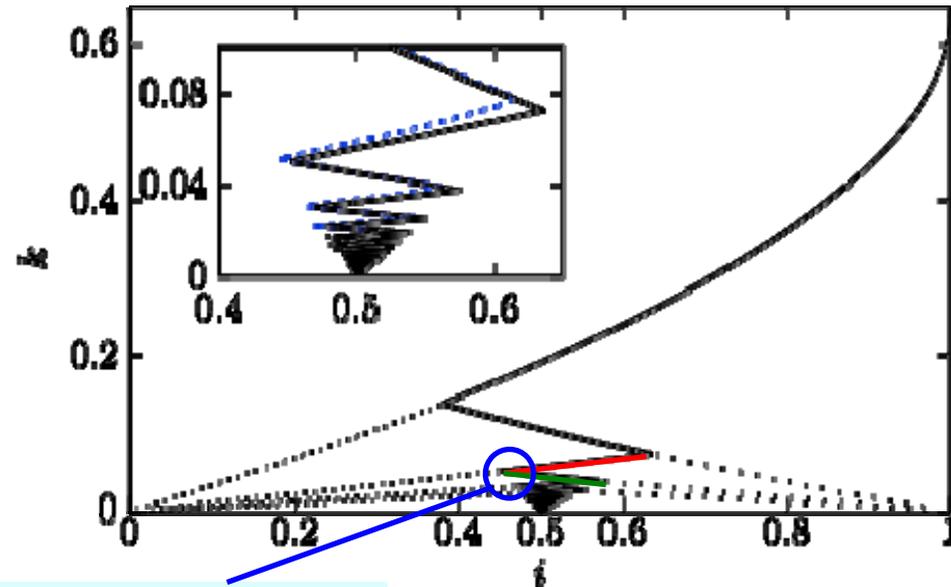
Realistic design:

- Two identical Josephson junctions, one of them shunted with large capacitance:



Measurements tuning external flux

- Building many SQUIDS with different L is tedious.
 - Tuning the **externally applied magnetic flux** Φ_e is easy.
- See one whole “tip” with a single device.

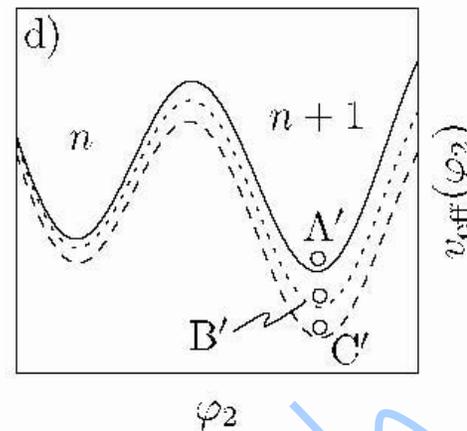
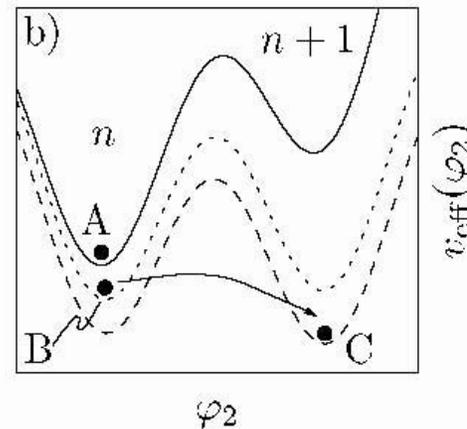
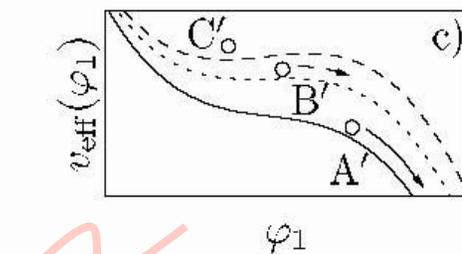
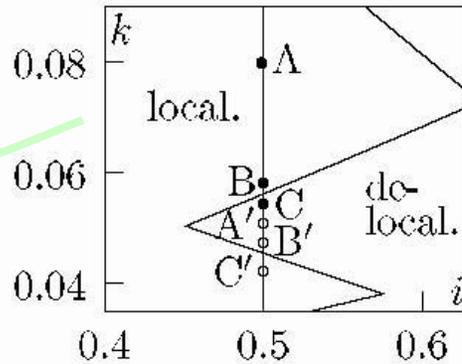
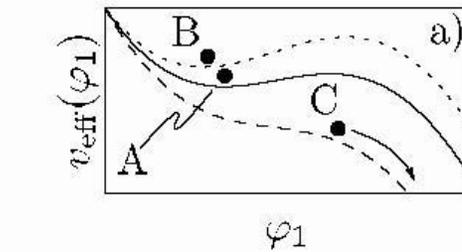


More insight

- A Heavy junction localized.
- B Barrier higher with lower k .
- C Barrier is lowered by force of light junction, heavy junction escapes.

Extract from phase diagram.

- A' Heavy junction delocalized
- B' Barrier higher with reduced k .
- C' Heavy junction is trapped again.



- A Light junction at rest in minimum n .
- B No change.
- C As minimum $n+1$ is lowered, it eventually tunnels.

- A' Light junction at rest in new global min. $n+1$.
- B' No change.
- C' No change.

Different cases

$$\alpha_2 = \eta_2 2\pi E_J / \hbar > 1.$$

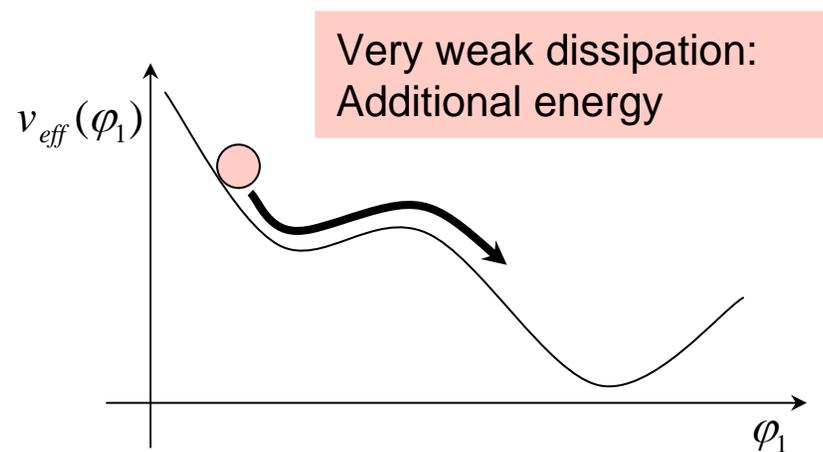
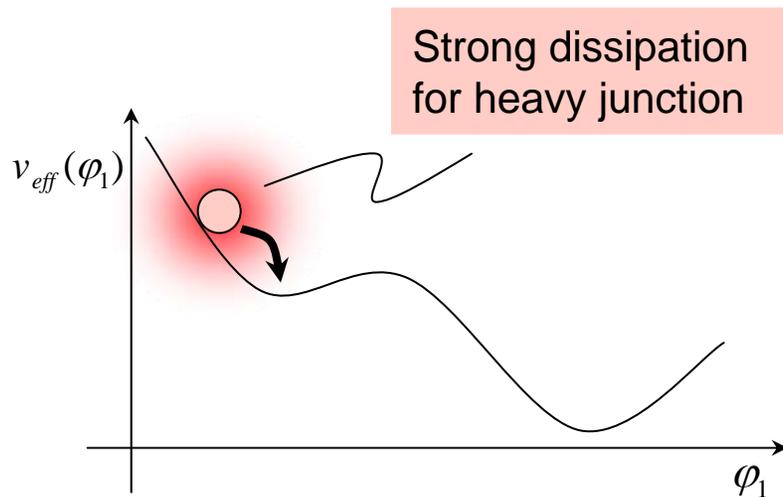
Strong dissipation for small junction.

• Strong damping of heavy junction:

- Total system is in given local minimum of 2d-SQUID potential.
- Large junction is delocalized if minimum becomes unstable.

For very weak damping of heavy junction:

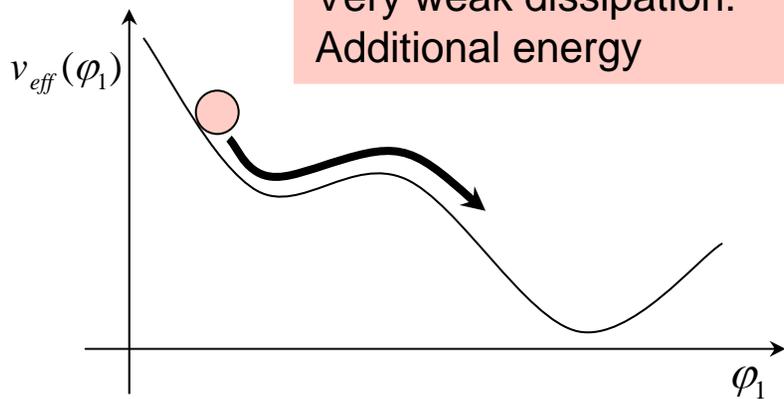
- Effective force of small junction lowers barrier for heavy junction,
- and shifts the local minimum.
- Heavy junction has additional initial energy (plasma oscillations), is delocalized at lower bias current.



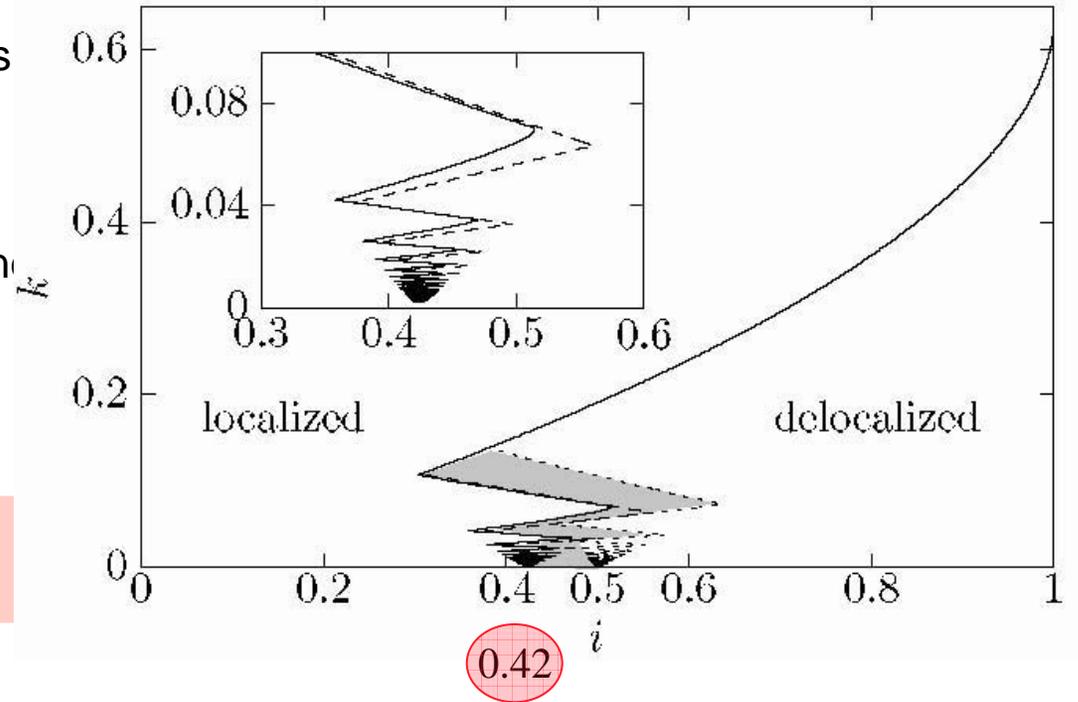
Strong dissipation for small junction.

For very **weak damping of heavy junction**:

- Effective force of small junction lowers barrier for heavy junction,
- and shifts the local minimum.
- Heavy junction has additional initial energy (**plasma oscillations**), is delocalized at lower bias current.



Very weak dissipation:
Additional energy



Summary and Outlook

SQUID with symmetric potential.

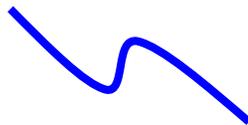


Junction 1 heavy/classical:
No tunneling.

Junction 2 light/quasi-
classical: Tunneling.



Small junction, after tunneling,
exerts effective force on heavy
junction.



For sufficiently high bias/sufficiently
low inductance: Total SQUID in finite
voltage state.

More general cases:

- Asymmetric critical currents.
- Geometrically asymmetric SQUIDS (asymmetric inductance).
- Additional external magnetic flux threading the ring (easily tunable parameter).

Other projects include:

- Detailed analysis of the low-dissipation case.