Elementary Charge Transfer Processes in a Superconductor Ferromagnet Multi-Terminal Structure

Wolfgang Belzig

Universität Konstanz, Germany

Collaborators/acknowledgement

Norwegian University of Technology, Trondheim

- **Jan Petter Morten**
- **Daniel Huertas-Hernando**
- **Arne Brataas**

Previous work with

- Johannes Boerlin, Christoph Bruder (Universität Basel)
- Yuli V. Nazarov (TU Delft)

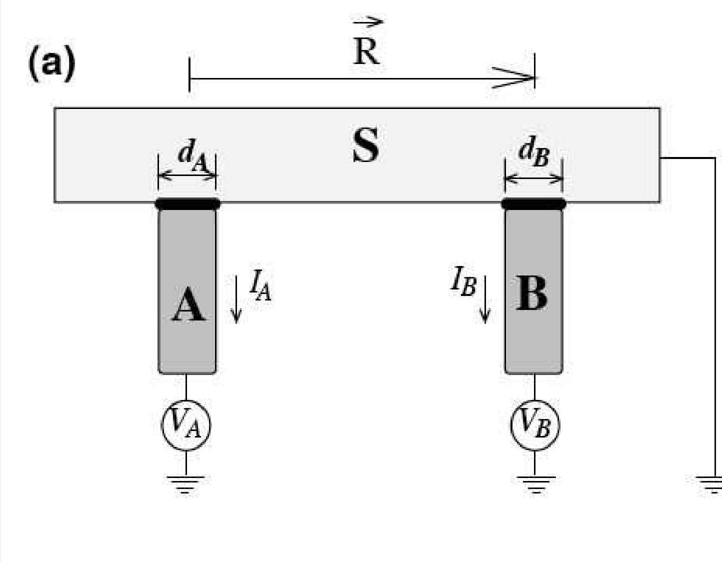
Financial support

- Centre of Advanced Study, Oslo
- Landesstiftung Baden-Württemberg: Kompetenznetzwerk Funktionelle Nanostrukturen
- Sonderforschungsbereiche 513 und 767 (DFG)
- Schwerpunktprogramm 1285 (DFG)

Content

- Nonlocal Andreev reflection
- Quasiclassical description of generic three terminal device with the quantum circuit theory
- Energy-dependence of nonlocal transport properties
- Complete characterization of transport by spin-resolved full counting statistics

Local vs. nonlocal transport processes



Spatially separated normal contacts

[Falci, Feinberg, Hekking, EPL 01]

- Andreev reflection (DA)
- Crossed Andreev reflection (CA)
- Elastic Cotunneling (EC)
  or Electron Transfer (ET)

Conductance matrix $(A = 1, B = 2)$:

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} G_{DA}^1 & G_{CA} - G_{EC} \\ G_{CA} - G_{EC} & G_{DA}^2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

Tunneling Hamiltonian: lowest order perturbative in transmission probabilities, linear response

$$G_{EC} \approx G_{AC}$$ in many channel limit

Not seen in experiment **[Russo et al., PRL 05]**

Effect of spin polarized contacts

Spin-dependent tunneling **[Falci, Feinberg,Hekking, EPL 01]**

Polarization of the contacts $P_{1/2} = \frac{D^{\uparrow}_{1/2} - D^{\downarrow}_{1/2}}{D^{\uparrow}_{1/2} + D^{\downarrow}_{1/2}}$

$D^{\sigma}_{1/2}$: spin-dependent density of states

$$\begin{pmatrix} G_{EC} \\ G_{CA} \end{pmatrix} \sim \begin{pmatrix} D_1^{\uparrow} D_2^{\uparrow} + D_1^{\downarrow} D_2^{\downarrow} \\ D_1^{\uparrow} D_2^{\downarrow} + D_1^{\downarrow} D_2^{\uparrow} \end{pmatrix} \sim \begin{pmatrix} 1 + P_1 P_2 \\ 1 - P_1 P_2 \end{pmatrix}$$

Conductances depend on the magnetic configuration

- Parallel ($P_1 = P_2$): G_{EC} enhanced, G_{CA} suppressed
- Antiparallel ($P_1 = -P_2$): G_{EC} suppressed, G_{CA} enhanced

Measurable through 'magnetoresistance':

$$G_{AP} - G_P \sim G_{EC} - G_{AC} \sim P_1 P_2$$

seen in experiment **[Beckmann et al., PRL 04]**

Current-current cross correlations

Tunneling Hamiltonian **[Bignon, Houzet, Pistolesi, Hekking, EPL 04]**

Current cross correlations from Poissonian statistics

$$S_{12} = 2eG_{CA}|V_1 + V_2| - 2eG_{EC}|V_1 - V_2|$$

Sign of cross correlations indicate type of process

positive = crossed Andreev

negative = elastic cotunneling

[see also Boerlin, Belzig, Bruder PRL 02, Samuelsson and Büttiker, PRL 02;]

Contact with cavity (non-tunneling, spin-polarized, incoherent)

Non Poissonian statistics results in cross correlation of arbitrary sign

antiparallel configuration -> adds positive cross correlations

parallel configuration -> adds negative cross correlations

[Sanchez, et al., PRB 03]

Circuit Theory Approach

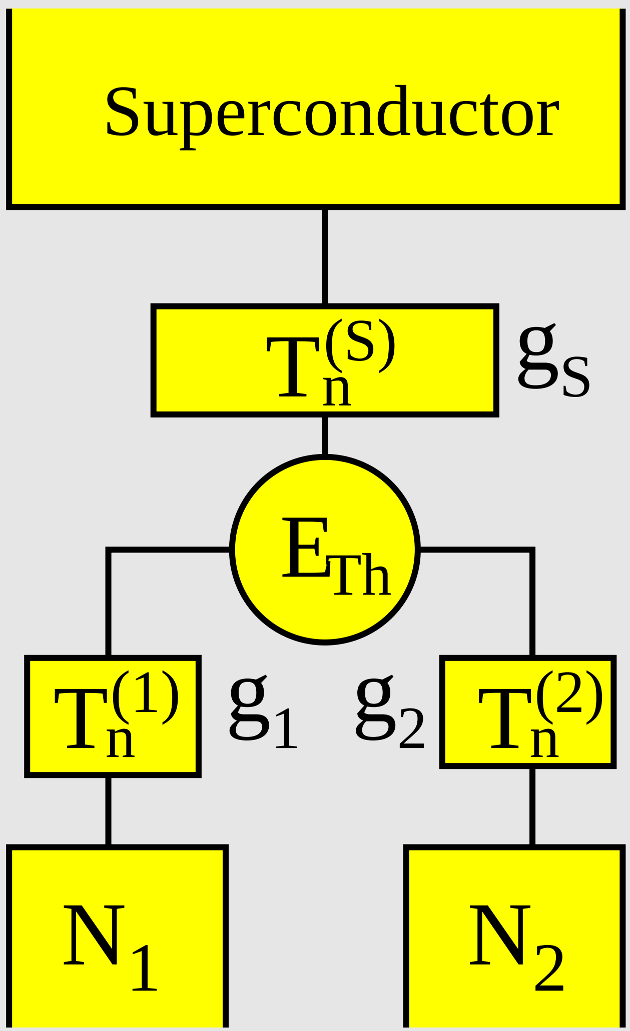
to

Crossed Andreev Reflection

What is the influence of non-tunneling contacts and/or finite bias?

Our model setup

Three terminal structure (one S and two N)



Main assumption:

- all leads are coherently coupled to central node
- connectors characterized by set of transmission eigenvalues $\{T_n^{(i)}\}, i = 1, 2, S$
- finite propagation time in node leads to Thouless energy $E_{Th} = \hbar/\tau_{dwell}$
- $E_{Th}, eV \ll \Delta \longrightarrow$ no quasiparticles tunneling into superconductor

[Morten, Brataas, Belzig, PRB 06]

Identification of contributions

Energy dependent spectral conductances

$$\begin{aligned} I_1(E) &= G_{\mathrm{ET}}(E)\,[f_2(E) - f_1(E)] \\ &\quad + 2G_{\mathrm{DA}}(E)\,[1 - f_1(E) - f_1(-E)] \\ &\quad + G_{\mathrm{CA}}(E)\,[1 - f_1(E) - f_2(-E)] \end{aligned}$$

Fermi factors indicate different processes:

- $f_2(E) - f_1(E)$: an electron tunneling from N_2 to N_1 $\longrightarrow$ electron transfer
- $1 - f_1(E) - f_1(-E)$: a pair tunneling from S to N_1 at E and $-E$ $\longrightarrow$ direct Andreev reflection
- $1 - f_1(E) - f_2(-E)$: a pair tunneling from S to N_1 at E and N_2 at $-E$ $\longrightarrow$ nonlocal Andreev reflection

Full Counting Statistics

approach to

Crossed Andreev Reflection

Can the spin-entanglement of

Cooper pairs

be seen in transport?

Full Counting Statistics (Generalities)

The probability to transfer of N charges in period t

$$P(N) \leftrightarrow S(\chi) = \ln \sum_N P(N) e^{iN\chi}$$

$S(\chi)$ is the cumulant generating function (CGF)

$$S(\chi) = M \ln \left[R + T e^{i\chi} \right] \quad \longrightarrow \quad \textbf{Binomial distribution}$$
$$S(\chi) = \bar{N} \left[e^{i\chi} - 1 \right] \quad \longrightarrow \quad \textbf{Poisson statistics}$$

General two-terminal quantum contact **[Levitov, Lesovik 93, ...]**

$$S(\chi) = \frac{eVt}{h} \sum_n \ln \left[1 + T_n (e^{i\chi} - 1) \right]$$

Multiterminal FCS: introducing multiple counting fields Example: multinomial distribution with M attempts

$$S(\{\chi_n\}) = M \ln \left[\sum_n p_n e^{i\chi_n} \right]$$

Full counting statistics and circuit theory

Circuit theory approach to FCS **[Nazarov, Ann. Phys. 99]**

- counting rotation of Keldysh space in terminals
  $\check{G}(\chi) = e^{i\chi\check{\tau}_K/2}\check{G}e^{-i\chi\check{\tau}_K/2}$
- CGF obtained from current $S(\chi) = \int d\chi \mathrm{Tr}\check{\tau}_K \check{I}(\chi)$

Other rules of circuit theory unchanged!

Applied to general quantum contact **[Belzig and Nazarv, PRL 01]**

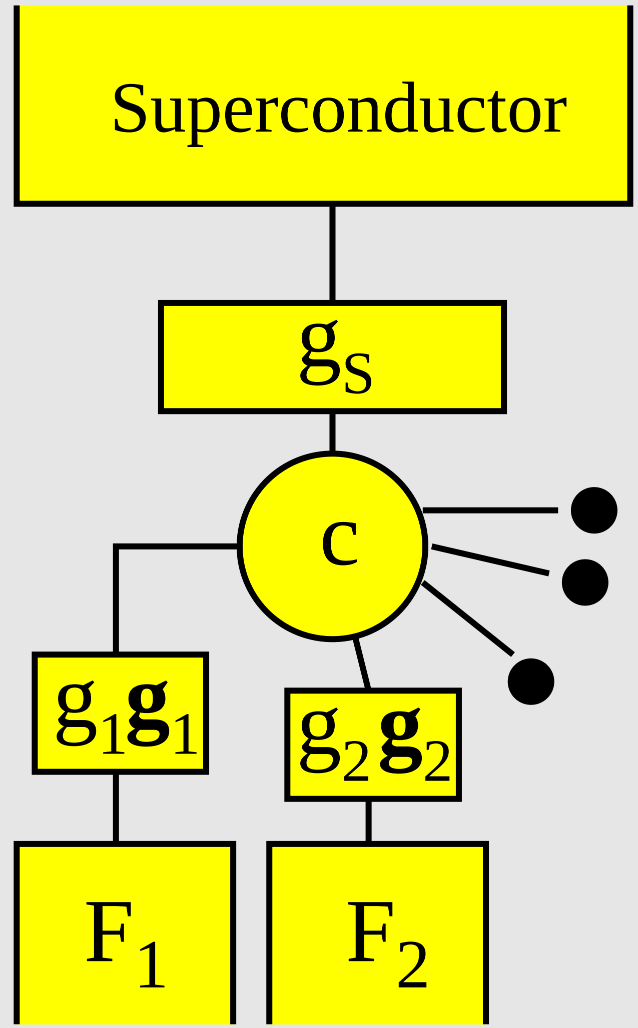
$$S(\chi) = \mathrm{Tr}\ln\left[1 + \frac{T}{4}\left(\{\check{G}_1(\chi), \check{G}_2\} - 2\right)\right]$$

used for SN contacts and SNS contacts
(multiple Andreev reflections **[see e.g. Cuevas and Belzig, PRL 04]**)

Multiterminal FCS can be calculated by introducing multiple counting fields χ_n. **[Bagrets and Nazarv, PRL 03]**

The superconductor-ferromagnet multiterminal device

- low temperature/bias voltage $k_B T, eV \ll E_{Th}$
- all tunnel contacts
- conductance between S and c: g_S
- conductance between F_n and c: g_n
- spin-dependent conductance $\vec{g}_n$ (encodes polarization $|\vec{g}_n|/g_n$ and magnetization direction $\vec{g}_n/|\vec{g}_n|$)
- total conductance to F $g = \sum_n g_n$
- total polarization $\vec{g} = \sum_n \vec{g}_n$



The matrix current $\check{I}_n = \frac{g_n}{2}\left[\check{G}_n, \check{G}_c\right] + \frac{1}{4}\left[\left\{\vec{g}_n \bar{\vec{\tau}} \hat{\tau}_3, \check{G}_n\right\}, \check{G}_c\right]$

[Huertas-Hernando, Belzig, Nazarov, PRL 02]

Full Counting Statistics

Calculated result for the CGF (with $g_\Sigma^2 = g_S^2 + g_F^2 + \vec{g}^2$)

$$S(\{\chi_n\}) = \frac{t_0 V g_\Sigma}{e\sqrt{2}} \times$$
$$\sqrt{g_\Sigma^2 + \sqrt{(g_S^2 - g_F^2 + \vec{g}^2)^2 + 4g_S^2(g_F^2 - \vec{g}^2)\sum_{mn} p_{mn} e^{i(\chi_m + \chi_n)}}},$$

Here

$$p_{mn} = \frac{g_m g_n - \vec{g}_m \vec{g}_n}{g^2 - \vec{g}^2}$$

correspond to the probabilities to detect one electron in F_n and one in F_m.

The interpretation of the FCS

The replacement $\sum_{mn} p_{mn} e^{i(\chi_m+\chi_n)} = e^{2i\chi}$ allows to factorize the probability into

$$P(\{N_n\}) = \int \frac{d^n\chi}{(2\pi)^n} e^{S(\{\chi_n\}) - i\sum_n \chi_n N_n}$$
$$= P\left(\{N_n\} \middle| \sum\nolimits_n N_n\right) P_S\left(\sum\nolimits_n N_n\right)$$

for $\sum_n N_n$ even and 0 otherwise.

- STEP 1: $P_S(2N)$ is probability to transfer N Cooper pairs
- STEP 2: $P(\{N_n\}|2N)$ is the conditional probability, that the $2N = \sum_n N_n$ electrons are detected in the terminals F_n

The CGF provides concrete expression for these probabilities

Cooper pair transfer probability

STEP 1:

CGF of Cooper pair transfer (obtained by setting all $\chi_n = \chi$)

$$S(\chi) = \frac{t_0 V}{\sqrt{2}e}\sqrt{g_\Sigma^2 + \sqrt{(g_S^2 - g^2 + \vec{g}^2)^2 + 4g_S^2(g^2 - \vec{g}^2)e^{i2\chi}}}$$

yields the total current/conductance

$$I = GV \; , \; G = \frac{g_S^2(g^2 - \vec{g}^2)}{\sqrt{g_S^2 + g^2}(g_S^2 + g^2 - \vec{g}^2)}$$

and the total shot noise (subpoissonian w.r.t. charge 2e)

$$S = 4eIF_{2e} \; , \; F_{2e} = 1 - \frac{(5(g_S^2 + g^2) - \vec{g}^2)(g^2 - \vec{g}^2)^2}{2(g_S^2 + g^2)((g^2 - \vec{g}^2) + g_S^2)^2}$$

Conditional detection probability

STEP 2: Probability to detect out of $\boldsymbol{N}$ Cooper pairs $\boldsymbol{N_1}$ electrons in $\boldsymbol{F_1}$, $\boldsymbol{N_2}$ in $\boldsymbol{F_2}$, etc. has the multinomial CGF

$$S_N(\chi) = 2N \ln \left[\sum_{mn} p_{mn} e^{i(\chi_m + \chi_n)} \right]$$

Detection probability $p_{mn} = \dfrac{g_m g_n - \vec{g}_m \vec{g}_n}{g^2 - \vec{g}^2}$

- independent of the coupling to superconductor g_S
- functional dependence is consistent with a pure spin-singlet density matrix for all magnetization configurations

  [di Lorenzo and Nazarov, PRL 05 (dLN)]
- spin accumulation does not disturb pure singlet character

  [in contrast to dLN, not understood yet!]

Nonmagnetic leads: $p_{mn} = \frac{g_m g_n}{g^2} = p_n p_m$

[Börlin, Belzig, Bruder, PRL 02]

Current and Noises in terms of probabilities

Current (into terminal F_n) with $p_n = \sum_m p_{nm}$

$$I_n = I p_n \quad I = GV$$

Current-current correlator (between F_n and F_m)

$$S_{nm} = 2eI\left[p_n \delta_{nm} + p_{nm} - 2(1 - F_{2e}) p_n p_m\right]$$

- direct access to p_{nm} through cross-correlators $S_{nm} (n \neq m)$
- dependence of p_{nm} on relative magnetization direction identifies spatially separated spin-singlets

Summary

What is the influence of non-tunneling contacts and finite bias?

- quasiclassical description of nonlocal conductance
- energy dependence on scale Thouless energy
- electron transfer dominates crossed Andreev reflection

Can the spin-entanglement of Cooper pairs be seen in transport?

- full counting statistics of SF-entangler
- interpretation as two step-process
- identification of singlets by probabilities

References

- Morten, Brataas, Belzig, PRB 06, Appl. Phys. A 07
- Morten, Huertas-Hernando, Brataas, Belzig, EPL 08